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Assessing the cyclical implications of IFRS 9 – a recursive model

by
Jorge Abad
Javier Suarez

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Executive summary

This occasional paper has been prepared to complement the mandate of the European Systemic Risk Board (ESRB) Task Force on the Financial Stability Implications of the Introduction of IFRS 9. It develops a recursive model to assess how different approaches to measuring credit impairment losses affect the average levels and dynamics of the impairment allowances associated with a bank's loan portfolio. The application of this model to a portfolio of European corporate loans suggests that IFRS 9 would tend to concentrate the impact of credit losses on profits and losses (P/L) and Common Equity Tier 1(CET1) capital at the very beginning of deteriorating phases of the economic cycle, which raises concerns about the procyclical effects of IFRS 9.

The analysis is based on a recursive model that contains the minimum elements needed to assess impairment allowances under IFRS 9 (EL^{IFRS9}) and alternative methods – incurred losses (IL), one-year expected losses (EL^{1Y}) and lifetime expected losses (EL^{LT}) – in a context where the differences between them have implications for both the average levels and the dynamics of the allowances associated with a given loan portfolio. The model is calibrated to analyse the behaviour of a typical portfolio of European loans over the business cycle.

The model builds on a simplified representation of a typical rating migration model, which has been augmented to incorporate the impact of aggregate risk (in the form of a state variable representing the expansion and the contraction phase of the business cycle). The model also takes into account the replacement of maturing loans with newly originated loans, endogenous loan rates and the impact of impairment allowances on P/L and the bank's capital position (summarised by its CET1 ratio). To capture the impact of business fluctuations, the analysis relies on evidence of the sensitivity of migration matrices and credit loss parameters to business cycles. It opts for a rather conservative perspective on the effects of the cycle on the relevant parameters (e.g. by abstracting from cyclical variation in loss given default (LGD) and in loan maturities) and considers the impact of average expansions and contractions only. As a result of cyclical variation in rating migration rates and probabilities of default (PD), the state of the business cycle causes significant variation in the composition of a bank's loan portfolio. During contractions, stage 2 loans (significantly deteriorated loans) and stage 3 loans (non-performing loans – NPLs) represent a larger share of the portfolio, and the realised yearly default rate (on performing loans) is more than twice as large as during expansions.

The results of the analysis point to relevant quantitative differences between the impairment allowances under IFRS 9 and those under the alternative measurement methods. There is also wide variation across aggregate states between the compared impairment measures. As shown in the paper, these differences produce changes in the model-implied dynamics of P/L, CET1, dividends and the average yearly frequency with which the bank needs to be recapitalised in order to comply with its minimum capital requirement.

On average, impairment allowances under IFRS 9 are about 152 basis points (of mean loan exposures) larger than under the incurred loss approach and about 88 basis points larger than under the one-year expected loss approach. They also vary more widely across aggregate states. In absolute terms, impairment allowances associated with stage 3 loans, followed by those associated with stage 2 loans, are the ones that contribute most to cross-state variation in impairment allowances. However, stage 3 loans are treated in the same way by all measures. The different cyclicality of the various measures therefore stems from the treatment of stage 1 and stage 2 loans, as well as the cyclical shift of loans across stages 1 and 2.

The results show that the most forward-looking impairment measures (EL^{IFRS9}, EL^{LT}) are the ones that generally make a bank more profitable and better CET1-capitalised during expansions and less profitable and less capitalised during contractions. This means that EL^{IFRS9} and EL^{LT} are the impairment measures that entail higher responsiveness to changes in economic conditions. Relative to IL (EL^{1Y}), the usage of EL^{IFRS9} implies an increase in the probability that a bank will need to be recapitalised during a contraction, from 10.3% (12.5%) to 14.9%. These differences are mirrored by a more modest increase, from 64.2% (67.1%) to 69.9%, in the probability of dividends being paid during an expansion.

The analysis also reveals that the more forward-looking methods, such as IFRS 9 (or the lifetime expected loss method envisaged by the Financial Accounting Standards Board (FASB) for the United States), imply sharper on-impact responses to changes in the aggregate state of the economy. Under the current calibration, the arrival of a typical recession implies an on-impact increase in impairment allowances, and the unfiltered effect of this on CET1 for a bank operating under the internal ratings-based (IRB) approach would be equivalent to about one-third of the bank's fully-loaded capital conservation buffer (CCB) and about twice as large as under the incurred loss approach. This means that the impact is sizeable (and larger than in the case of other impairment measures) but also suitably absorbable if the CCB is available and able to be effectively used when the shock hits.

Several extensions of the model, which are also discussed in the paper, show that the results are qualitatively and (in relative terms) quantitatively very similar for a bank following the standardised approach to capital requirements and that the arrival of a contraction that is anticipated to be more severe or longer than average will tend to produce sharper responses. By contrast, if the arrival of a contraction can be anticipated one period in advance, its impact will tend to be smaller.

Overall, and subject to the caveats and limitations derived from the simplifying modelling assumptions (e.g. abstracting from differences in lending behaviour or ex ante precautions induced by each of the approaches to impairment measurement) and the reliance on historical data, the results of the analysis mean that it cannot be ruled out that, contrary to its intended purpose, IFRS 9 in certain circumstances amplifies rather than reduces the variability in capital pressures over the business cycle, with potential well-known implications for the cyclicality of credit supply.

Section 1
Introduction

This paper develops a recursive model for assessing the implications of the new approach to measuring credit impairment losses established by IFRS 9, the new international standard for the valuation of financial assets and liabilities, which will come into force in January 2018.[1] The key innovation of IFRS 9 is a shift from an incurred loss approach to an expected loss approach. Under IFRS 9, impairment allowances will be calculated using two projection horizons. For exposures that have not suffered a significant increase in credit risk, impairment allowances will equal the one-year expected losses discounted at the effective contractual interest rate. For exposures that have suffered a significant deterioration in credit quality, impairment allowances will equal the lifetime expected losses, also discounted at the effective contractual interest rate.

The recursive model described in this paper contains the minimum elements needed to assess impairment allowances under the above-mentioned and alternative methods, in a context where the differences between those methods have implications for both the average levels and the dynamics of the allowances associated with a given loan portfolio. The model is calibrated to analyse the behaviour of a typical portfolio of European loans over the business cycle, in order to assess the potential implications of the new approach to measuring impairments on the dynamics of banks' P/L and CET1.

One difficulty in modelling the measurement method proposed by IFRS 9 is keeping track not only of the credit quality of a given loan but also of its credit quality at origination and its effective contractual interest rate. This difficulty introduces high dimensionality to the state space required to describe the evolution over time of a loan portfolio in a compact way. In general, a cohort of loans of a given rating, even if assumed to be composed of ex ante identical loans with the same effective contractual rate and to have a credit quality that evolves according to a cohort-independent rating migration matrix, would have to be distinguished from a cohort of loans originated with different effective contractual rates, even if their origination rating were the same.

Ideally, one would like to characterise the performance of alternative credit allowance measurement methods in a set-up where the pricing of the loans and the dynamics of the composition of the portfolio of loans of a representative holder (e.g. a bank) could be endogenously established in a way consistent with the background assumptions regarding the rating migration matrix, the LGD parameters and the maturity of the loans, as well as the evolution of aggregate risk and its impact on the aforementioned parameters. Ideally, one would like to be explicit about newly originated loans that enter the portfolio, possibly replacing loans that mature or that are resolved.

In a stationary situation without aggregate risk, one would like to be able to obtain the ergodic distribution of loans over the categories relevant for the measurement of their credit loss allowances under IFRS 9 and alternative methods. One would also like to be able to characterise the system's dynamic response to shocks that either perturb punctually the composition of the loan portfolio (like the unanticipated once-and-for-all shocks commonly analysed in macroeconomic theory) or that affect the dynamics of the system more recurrently, in the form of systematic aggregate risk. Moreover, one would like to keep the model sufficiently rich to be suitable for calibration, i.e. for providing a tentative quantitative (and not only qualitative) assessment of the implications of IFRS 9 compared with other methods for measuring credit loss allowances.

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[1] See IASB (2014).

We achieve all this using a simple recursive rating migration model, which is highly tractable thanks to a rather compact description of possible credit risk categories and, in the version with aggregate risk, a stylised description of the economic cycle as a two-state Markov chain.[2] A shortcut which simplifies the process is the modelling of loan maturity as random (as in Leland and Toft (1996)), which means we do not have to keep track of loan vintages.[3]

We calibrate the versions of the model with and without aggregate risk in order to match the characteristics of a typical portfolio of corporate loans issued by European banks. In the version with aggregate risk, we use evidence of the sensitivity of migration matrices and credit loss parameters to business cycles, as in Bangia et al. (2002). The results point to relevant differences between IFRS 9 and alternative measurement methods (incurred loss, one-year expected loss and lifetime expected loss) regarding the level of the allowances and their dynamic responses to shocks.[4] More forward-looking methods, such as IFRS 9 (or the lifetime expected loss method envisaged by the FASB for the United States) imply significantly larger impairment allowances and sharper on-impact responses to negative shocks to (expected) credit quality, including those associated with changes in the aggregate state of the economy.

Under the current calibration of the model with aggregate risk, the arrival of a typical recession implies an on-impact increase in IFRS 9 impairment allowances, of which the unfiltered effect on CET1 would be equivalent to about one-third of a bank's fully-loaded CCB. This means that the impact is sizeable, but also suitably absorbable if the buffer is available when the shock hits. As we show, the arrival of a contraction that is anticipated to be more severe or longer than average will tend to produce sharper responses. By contrast, if the arrival of a contraction can be anticipated one period in advance, its impact will tend to be smaller.

These results suggest that, if regulatory filters do not offset or smooth out the cyclical impact of impairment allowances on CET1, IFRS 9 may mean that banks experience more sudden falls in regulatory capital at the very end of expansionary phases of the credit or business cycle. Banks can, of course, try to prepare for this by holding higher precautionary capital buffers during good times. Alternatively, they may adjust, when the time comes, by cutting dividends or by issuing new equity, although there is ample anecdotal evidence and some formal empirical evidence to indicate that, when confronted with such choices, banks undertake at least part of the adjustment by reducing their risk-weighted assets (RWAs), for example by cutting the origination of new loans or rebalancing towards safer ones.[5] In this case, IFRS 9 might imply negative feedback effects on the supply of credit just before the cycle starts to deteriorate, very much via the same type of mechanisms extensively discussed in the literature on the procyclical effects of risk-sensitive bank capital requirements and the countercyclical effects of dynamic provisions.[6] We cannot therefore

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[2] See Trueck and Rachev (2009) as a general reference, and Gruenberger (2012) for an early application to the analysis of IFRS 9.

[3] Instead, in the version with aggregate risk, we need to keep track of the aggregate state of the economic cycle in which the loans are originated, since this affects the interest rate relevant for the discounting of the expected credit losses.

[4] Each of the alternative methods can be associated with existing or forthcoming accounting practices. Incurred loss was the standard under the current IAS 39, US GAAP and most other national GAAPs. One-year expected loss is the method behind the internal-ratings based approach to capital requirements. Lifetime expected loss is the method envisaged by the FASB as a replacement for the incurred loss method in the United States.

[5] See, for example, Mésonnier and Monks (2015), and Gropp et al. (2016), as well as the references therein. The evidence in the latter paper is consistent with average bank responses to the ESRB questionnaire on assessing second-round effects that accompanied the EBA stress test in 2016. The questionnaire examined the way in which banks would expect to re-establish their desired levels of capitalisation after exiting the adverse scenario.

[6] Contributions to the literature on the procyclical effects of capital requirements include Kashyap and Stein (2004), and Repullo and Suarez (2013). Jiménez et al. (2017) document the countercyclicality associated with the Spanish statistical provisions, with results suggesting that the effects of changes in capital pressure on credit are significantly more pronounced in recessions than in expansions.

rule out that, contrary to its intended purpose, IFRS 9 amplifies rather than reduces the cyclicality of credit supply.

From a normative perspective, this potential shortcoming of the new approach should be weighed against the benefits of creating provisions for future credit losses earlier and more cautiously, which include having financial statements that reflect the weakness or strength of the reporting institution in a more timely and reliable way.[7] Offering such a comprehensive assessment exceeds the scope of this paper. The results presented should therefore not be interpreted as a comprehensive assessment of the benefits and costs of IFRS 9, but as a first quantitative analysis of its potential procyclical effects. This analysis may be useful in the context of discussions on adjustments that may need to be made to microprudential regulation or macroprudential policies in the light of the new accounting standards (see, for example, Basel Committee on Banking Supervision (2016)).

The paper is organised as follows. Section 2 describes the baseline model without aggregate risk. Section 3 develops the formulae for measuring impairment losses under the various approaches that we compare, as well as formulae for assessing their effects on P/L and CET1. Section 4 explores the effect of an ad hoc shock to the credit quality of bank loans in the calibrated version of the baseline model. Section 5 presents and calibrates the model with aggregate risk and uses it to analyse the response to the arrival of a typical recession under the various measures. Having looked at banks operating under the IRB approach to capital requirements as a benchmark, Section 6 analyses the results in the case of a bank operating under the standardised approach. Section 7 describes several extensions. Section 8 discusses the macroprudential implications of the results. Section 9 concludes the paper.

[7] See Laeven and Majnoni (2003), and Huizinga and Laeven (2012) for evidence of bank provisioning practices and a discussion of their implications. See Basel Committee on Banking Supervision (2015) for a literature review.

Section 2
Baseline model without aggregate risk

This section develops a simple recursive model of a bank's loan portfolio. In later sections, we will derive formulae and other elements necessary for measuring credit impairments under the various methods that we aim to compare, and formulae for assessing their impact on P/L and CET1. The model is based on ten assumptions that fully describe the elements needed to understand the dynamics of loan origination, rating migration, default, maturity and pricing at origination of the loans that make up the loan portfolio. The tree in Figure 1 summarises the relevant contingencies over the life of a loan (variables on each branch describe the relevant marginal conditional probabilities).

Model assumptions:

1. In each date t, existing loans belong to one of three credit rating categories: standard (j=1), substandard (j=2) or non-performing (j=3). We denote the measure of loans that belong to each category as x_{jt}.
2. In each date t, the bank originates a continuum of standard loans of measure $e_{1t} > 0$, with a principal normalised to one and a constant interest payment per period equal to c. In the language of IFRS 9, c is the effective contractual interest rate at which future expected losses will be discounted. In the analysis of steady states, we will assume a steady flow of entry of new loans $e_{1t} > e_1$ at each t.
3. Each loan's exposure at default is constant and equal to one up to maturity.
4. Loans mature randomly and independently. Specifically, loans rated j=1,2 mature at the end of each period with a constant probability δ_j.[8] This implies that, conditional on remaining in rating j, a loan's expected life span is $1/\delta_j$ periods. By the law of large numbers, the fraction of loans of a given rating j that mature at the end of each period is δ_j. In steady state, this produces a stream of maturity cash flows very similar to those that would emerge in the case of a portfolio of perfectly staggered loans with identical deterministic maturities at origination.
5. In the case of NPLs (j=3), δ_3 represents the independent per-period probability of a loan being resolved, in which case it pays back a fraction $1 - \lambda$ of its principal and exits the portfolio. The constant λ is therefore the loss rate at resolution, which coincides with the loan's expected LGD in the baseline model.
6. Each loan rated j=1,2 at t that matures at $t + 1$ defaults independently with probability PD_j. Maturing loans that do not default pay back their principal of one plus interest c. Each defaulted loan is resolved within the same period with an independent probability $\delta_3/2$.[9] Otherwise, it enters the stock of NPLs (j=3).
7. Each loan rated j=1,2 at t that does not mature at $t + 1$ goes through one of the following exhaustive possibilities:

[8] Allowing for $\delta_1 \neq \delta_2$ may help capture the possibility that longer maturity loans get early redeemed with different probabilities depending on their credit quality.

[9] We divide δ_3 by two to reflect the fact that, if loans default uniformly during the period between t and $t + 1$, they will, on average, have just half a period to be resolved. The model can trivially accommodate alternative assumptions on same-period resolutions.

(a) Default, which occurs independently with probability PD_j. As is the case when a maturing loan defaults, a non-maturing loan that defaults is resolved within the same period with probability $\delta_3/2$, yielding $1-\lambda$. Otherwise, it enters the stock of NPLs (*j*=3).

(b) Migration to rating $i \neq j$ (*i*=1,2), which occurs independently with probability a_{ij}. In this case, the loan pays interest c and continues for one more period with its new rating.

(c) Staying in rating j, which occurs independently with probability $a_{jj} = 1 - a_{ij} - PD_j$. In this case, the loan pays interest c and continues for one more period with its previous rating.

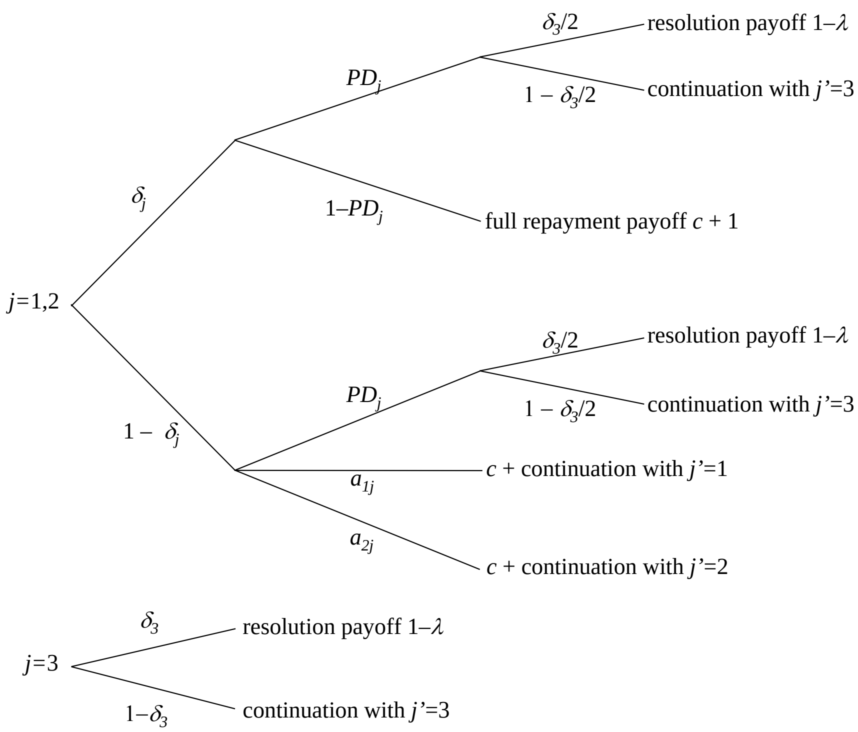
8. NPLs (*j*=3) pay no interest and never return to the performing categories. They accumulate in category *j*=3 up to their resolution.[10]

9. The contractual interest rate c is established at origination as in a perfectly competitive environment with risk-neutral banks that have an opportunity cost of funds between any two periods equal to a constant r. The originating bank is assumed to hold the loans up to their maturity, hence satisfying the "business model" condition required by IFRS 9 for the valuation of basic lending assets at amortised cost.

10. Finally, one period corresponds to a calendar year, and dates t, $t+1$, $t+2$, etc. denote end-year accounting reporting dates (so "period t" ends at "date t").

Figure 1
Possible transitions of a loan rated *j*

(possible contingencies between two dates and their implications for pay-offs and continuation value)



[10] For calibration purposes, it is possible to account for potential gains from the unmodelled interest accrued while in default or from returning to performing categories by adjusting the loss rate λ.

In the version of the model with aggregate risk that we present in Section 5, we will allow all the parameters in the tree depicted in Figure 1 to vary with the aggregate state of the economy.

2.1 Portfolio dynamics without aggregate risk

The model presented so far has no aggregate risk. By the law of large numbers, the evolution of the loans in each rating category can be represented by the difference equation:

$$x_t = Mx_{t-1} + e_t \tag{1}$$

where

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix} \tag{2}$$

is the vector that describes the loans in each rating category $j = 1,2,3$;

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} (1-\delta_1)a_{11} & (1-\delta_2)a_{12} & 0 \\ (1-\delta_1)a_{21} & (1-\delta_2)a_{22} & 0 \\ (1-\delta_3/2)PD_1 & (1-\delta_3/2)PD_2 & (1-\delta_3) \end{pmatrix} \tag{3}$$

accounts for the migrations across categories of the non-matured, non-defaulted loans and

$$e_t = \begin{pmatrix} e_{1t} \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

accounts for the new loans originated at each date, which we write to reflect the fact that, at origination, all loans have rating $j = 1$.

2.2 Steady state portfolio without aggregate risk

If the amount of newly originated loans is equal at all dates ($e_t = e$ for all t), the loan portfolio will asymptotically converge to a time-invariant or steady-state portfolio x^* that can be obtained as the vector that solves

$$x = Mx + e \Leftrightarrow (I - M)x = e \tag{5}$$

that is,

$$x^* = (I - M)^{-1}e. \tag{6}$$

Section 3
Measuring impairment losses

In this section, we derive formulae for measuring the impairments generated by the previously described loan portfolio under different approaches. We also discuss how to endogenously determine a contractual loan rate c consistent with our assumptions on the competitive pricing of loans at origination. Finally, we provide formulae for assessing the impact of impairment measurement on a bank's P/L and CET1.

3.1 Incurred losses

The incurred loss approach has characterised accounting standards in most jurisdictions in recent years. By January 2018 IFRS 9 is scheduled to replace it in all jurisdictions that have the International Accounting Standards Board (IASB) as an accounting standards setter.

Under the narrowest interpretation, allowances measured on an incurred loss basis are restricted to NPLs. In our set-up, the incurred losses reported at t are

$$IL_t = \lambda x_{3t}, \tag{7}$$

since the loss rate λ is the expected LGD of the bank's NPLs at date t. Note that, under our assumptions, the losses associated with loans that default between dates $t-1$ and t which are resolved within such period, $\lambda(\delta_3/2)(PD_1 x_{1t-1} + PD_2 x_{2t-1})$, do not enter IL_t and therefore will be directly recorded in the P/L of year t.

3.2 Discounted one-year expected losses

The one-year expected loss approach is, roughly speaking, the approach prescribed for regulatory purposes for banks following the IRB approach to capital requirements.[11]

For loans performing at t, impairment allowances are measured on a discounted one-year expected loss basis. They are therefore forward-looking, but the forecasting horizon is limited to one year. For loans with j=1,2, the allowance is calculated taking into account the losses due to default events that are expected to occur within the immediately upcoming year. For NPLs (j=3), the allowance equals the whole (non-discounted) LGD of the loans, so we can write

$$EL_t^{1Y} = \lambda[\beta(PD_1 x_{1t-1} + PD_2 x_{2t-1}) + x_{3t}] \tag{8}$$

where $\beta = 1/(1+c)$ is the discount factor based on the contractual interest rate c. In Section 3.5, we derive an expression for the endogenous value of c consistent with our assumptions on loan pricing.

In matrix notation, which will be useful when extending the loss forecasting horizon to several years, the above credit loss allowances can also be expressed as

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[11] In fact, Basel Committee on Banking Supervision (BCBS) prescriptions on regulatory provisions establish that the PDs that must feed into the above formula must be through-the-cycle (rather than point-in-time) estimates of the corresponding PD. By the same logic, they establish that the LGDs must conservatively reflect a distressed liquidation scenario rather than a central scenario. Prescriptions for discounting are also slightly different. To simplify the analysis, we abstract from all these differences.

$$EL_t^{1Y} = \lambda[\beta b x_t + x_{3t}], \quad (9)$$

where

$$b = (PD_1, PD_2, 0). \quad (10)$$

3.3 Discounted lifetime expected losses

The lifetime expected loss approach is the approach that the FASB, the US accounting standards setter, envisages as a replacement for the current incurred loss approach under the US generally accepted accounting principles (GAAP).

For loans performing at t, credit loss allowances under the lifetime expected loss approach are the sum of the discounted expected losses that the loans are projected to cause in each future year. However, for NPLs, the allowance covers the whole (non-discounted) LGD of the affected loans. The allowances can therefore be expressed as

$$EL_t^{LT} = \lambda b(\beta x_t + \beta^2 M x_t + \beta^3 M^2 x_t + \beta^4 M^3 x_t + \ldots) + \lambda x_{3t}, \quad (11)$$

which reflects the fact that the losses expected from currently performing loans at any future date $t+\tau$ can be found as $\lambda b M^{\tau-1} x_t$, where b contains the relevant one-year-ahead PD (see (10)) and $M^{\tau-1} x_t$ gives the projected composition of the portfolio at each future date $t+\tau-1$. [12]

Equation (11) can be re-expressed as

$$EL_t^{LT} = \beta \lambda b(I + \beta M + \beta^2 M^2 + \beta^3 M^3 + \ldots) x_t + \lambda x_{3t} \quad (12)$$

where the parenthesis contains the infinite sum of a geometric series of matrices, which can be expressed as

$$B = (I - \beta M)^{-1}. \quad (13)$$

Thus, we can calculate EL_t^{LT} using

$$EL_t^{LT} = \lambda(\beta b B x_t + \lambda x_{3t}). \quad (14)$$

Obviously, $B \geq I$, so $EL_t^{LT} \geq EL_t^{1Y}$.

3.4 Discounted expected losses under IFRS 9

As already mentioned, for performing loans, IFRS 9 adopts a mixed-horizon approach that combines the one-year-ahead and lifetime approaches described above. Specifically, it applies the one-year-ahead measurement to loans that have not suffered a significant increase in credit risk since origination ("stage 1" loans), which for us are the standard loans x_{1t}. It applies the lifetime measurement to performing loans with deteriorated credit quality ("stage 2" loans), which for us are

12 In the FASB proposal, the discount factor β is not based on the effective contractual interest rate of the loan, but on a reference risk-free rate. However, we will abstract from this feature and use a common definition of β throughout all of the impairment measures compared in this paper.

the substandard loans x_{2t}. Finally, for NPLs ("stage 3" loans), x_{3t}, the allowance simply equals the whole (non-discounted) expected LGD, as under any of the other approaches.

Combining the formulae obtained in (9) and (14), the impairment allowances under IFRS 9 can be described as

$$EL_t^{IFRS9} = \lambda\beta b \begin{pmatrix} x_{1t} \\ 0 \\ 0 \end{pmatrix} + \lambda\beta bB \begin{pmatrix} 0 \\ x_{2t} \\ 0 \end{pmatrix} + \lambda x_{3t}, \quad (15)$$

which, together with $EL_t^{LT} \geq EL_t^{1Y}$, implies that $EL_t^{1Y} \leq EL_t^{IFRS9} \leq EL_t^{LT}$.

3.5 Loan pricing

Taking advantage of the recursive nature of the model, we can obtain the bank's ex-coupon value of loans rated j at any given date, v_j, by solving the following system of Bellman-type equations:

$$v_j = \mu\left[(1 - PD_j)c + (1 - PD_j)\delta_j + PD_j(\delta_3/2)(1 - \lambda) + m_{1j}v_1 + m_{2j}v_2 + m_{3j}v_3\right], \quad (16)$$

for j=1,2, and

$$v_3 = \mu[\delta_3(1 - \lambda) + (1 - \delta_3)v_3], \quad (17)$$

where $\mu = 1/(1 + r)$ is the discount factor of the risk-neutral bank and the square brackets in (16) and (17) contain the continuation pay-offs or value obtained in the contingencies that, in each case, can occur one period ahead (weighted by the corresponding probabilities).[13]

In (16), the first term in the square brackets represents the interest that the loan currently rated j will pay at the next due date if it continues to perform. The second term captures the terminal value obtained if the loan matures without defaulting. The third term represents the terminal value recovered if the loan defaults and is resolved within the period. The fourth and fifth terms reflect the continuation value obtained if the loan does not mature and receives (or retains) rating 1 or 2, respectively, for the next period. The last term measures the continuation value obtained if the loan defaults but is not resolved within the period, thus becoming an NPL.

In (17), the first term in the square brackets represents the terminal value recovered if an NPL is resolved within the period. The last term reflects the continuation value of the NPL if it remains unresolved at $t + 1$.

Perfect competition implies that the value of extending a loan of size one rated as standard (j=1) at origination must equal the value of its principal (one), so that the bank obtains zero net present value from its origination. Solving for c delivers the endogenous contractual interest rate that enters the discount factor $\beta = 1/(1 + c)$ used in the various expectation-based impairment measures mentioned above.

[13] For calibration purposes, the discount rate r does not need to equal the risk-free rate. It is possible to adjust the value of r to reflect the bank's marginal weighted average costs of funds or even an extra element capturing (in reduced form) a mark-up applied to that cost if the bank is not perfectly competitive.

3.6 Implications for P/L and CET1

To explore the implications of impairment measurement for the dynamics of the P/L account and for CET1, we need to make further assumptions regarding the bank holding the loan portfolio discussed so far and its capital structure. To simplify the discussion, we abstract from bank failure and assume that the bank's only assets at the end of any period t are the loans described by vector x_t, and that its liabilities consist exclusively of (i) (risk-free) one-period debt, d_t, that promises to pay interest r per period; (ii) impairment allowances a_t computed under one of the measurement approaches described above (so $a_t = IL_t, EL_t^{1Y}, EL_t^{IFRS9}, EL_t^{LT}$); and (iii) CET1, k_t. This means that the bank's balance sheet at the end of any period t can be described as

$$\begin{array}{c|c} \hline x_{1t} & d_t \\ x_{2t} & a_t \\ x_{3t} & k_t \end{array} \qquad (18)$$

with the law of motion of x_t described by (1) and the law of motion of k_t given by

$$k_t = k_{t-1} + PL_t - \mathrm{div}_t + \mathrm{recap}_t, \qquad (19)$$

where PL_t is the result of the P/L account at the end of period t, $\mathrm{div}_t \geq 0$ are cash dividends paid at the end of period t, and $\mathrm{recap}_t \geq 0$ are injections of CET1 at the end of period t. Under these assumptions, the dynamics of d_t can be recovered residually from the balance sheet identity, $d_t = \Sigma_{j=1,2,3} x_{jt} - a_t - k_t$.

The result of the P/L account can, in turn be, written as

$$PL_t = \left\{\sum\nolimits_{j=1,2} \left[c\left(1 - PD_j\right) - \frac{\delta_3}{2} PD_j \lambda\right] x_{jt-1} - \delta_3 \lambda x_{3t-1}\right\} - r\left(\sum\nolimits_{j=1,2,3} x_{jt-1} - a_{t-1} - k_{t-1}\right) - \Delta a_t, \qquad (20)$$

where the first term contains the income from performing loans net of realised losses on defaulted loans resolved during period t, the second term is the interest paid on d_t and the third term is the variation in credit loss allowances between periods $t-1$ and t.To model dividends, div_t, and equity injections, recap_t, in a simple manner, we assume that the bank manages the evolution of its CET1 using a simple sS-rule based entirely on existing capital regulations.[14] Specifically, current Basel III prescriptions include the minimum capital requirement and the CCB. Minimum capital requirements force the bank to operate with a CET1 of at least $\underline{k}_t$, while the CCB requires the bank to retain profits, whenever feasible, until it has a fully-loaded buffer equal to 2.5% of its RWAs. This means that a bank with positive profits must accumulate them until its CET1 reaches a level $\overline{k}_t = 1.3125\underline{k}_t$.[15]

Thus, we assume the bank's dividends and equity injections to be determined as

$$\mathrm{div}_t = \max\left[(k_{t-1} + PL_t) - 1.3125\underline{k}_t, 0\right] \qquad (21)$$

and

[14] This rule can be rationalised as the one that minimises the equity capital committed to support the loan portfolio. Its working here is consistent with the absence of fixed costs associated with the raising of new equity. If such costs were to be introduced, the optimal rule would imply discrete recapitalisations to a level within the two bands if the lower band were to be otherwise passed, as in Fischer, Heinkel, and Zechner (1989).

[15] Under Basel III, RWAs equal 12.5 (or 1/0.08) times the bank's minimum required capital $\underline{k}_t$. Thus a fully-loaded CCB amounts to a multiple of $0.025 \times 12.5 = 0.3125$ of $\underline{k}_t$.

$$\text{recap}_t = \max\left[\underline{k}_t - (k_{t-1} + PL_t), 0\right], \quad (22)$$

respectively.

Minimum capital requirement under the IRB approach

For banks or portfolios operating under the IRB approach, the IRB formula specified in BCBS (2004, paragraph 272) establishes that the regulatory capital requirement must be

$$\underline{k}_t^{IRB} = \sum\nolimits_{j=1,2} \gamma_j x_{jt}, \quad (23)$$

with

$$\gamma_j = \lambda \frac{1 + \left[(1/\delta_j) - 2.5\right] m_j}{1 - 1.5 m_j} \left[\Phi\left(\frac{\Phi^{-1}(PD_j) + \text{cor}_j^{0.5}\Phi^{-1}(0.999)}{(1 - \text{cor}_j)^{0.5}}\right) - PD_j\right], \quad (24)$$

where $m_j = [0.11852 - 0.05478\ln(PD_j)]^2$ is a maturity adjustment coefficient, $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution and cor_j is a correlation coefficient fixed as $\text{cor}_j = 0.24 - 0.12(1 - \exp(-50PD_j))/(1 - \exp(-50))$.

Minimum capital requirement under the standardised approach (SA)

For banks or portfolios operating under the SA, the regulatory minimum capital requirement applicable to loans to corporations without an external rating is just 8% of the exposure net of its "specific provisions" (a regulatory concept related to impairment allowances). Assuming that all the loans in x_t correspond to unrated borrowers and that all the impairment allowances qualify as specific provisions, this implies that

$$\underline{k}_t^{SA} = 0.08\left(\sum\nolimits_{j=1,2,3} x_{jt} - a_t\right). \quad (25)$$

Formulae (23) and (25) will allow us to assess the impact of different impairment measurement methods on the dynamics of PL_t, k_t, div_t, and recap_t under each of the approaches to capital requirements.

It is important to note that, as a first approximation, our analysis abstracts from the existence of "regulatory filters" dealing with the implications of possible discrepancies between "accounting" and "regulatory" provisions and their effects on "regulatory capital". In this sense, our assessment can be seen as an evaluation of the impact of accounting rules on bank capital dynamics in the extreme event that bank capital regulators accept the new accounting provisions (and the resulting accounting capital) as provisions (and available capital) for regulatory purposes as well.[16]

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16 In the case of banks operating under the IRB approach, the current regulatory regime (which might be revised to accommodate IFRS 9) defines regulatory provisions as one-year expected losses, EL_t^{1Y}. If EL_t^{1Y} exceeds the accounting allowances, a_t, the difference, $EL_t^{1Y} - a_t$, must be subtracted from CET1. By contrast, if $EL_t^{1Y} - a_t < 0$, the difference can be added back as Tier 2 capital up to a maximum of 0.6% of the bank's credit RWAs. In the case of banks operating under the SA approach, there is a filter for general provisions (which, for simplicity's sake, we assume to be zero in our analysis), which can be added back as Tier 2 capital up to a maximum of 1.25% of credit RWAs.

Section 4
An initial quantitative exploration

The model described so far features a relatively small number of parameters. Table 1 describes their value under a parameterisation intended to represent a typical portfolio of corporate loans issued by European Union (EU) banks. Given the absence of detailed publicly available microeconomic information on such portfolios, the calibration relies on matching aggregate variables taken from recent European Banking Authority (EBA) reports and European Central Bank statistics using rating migration and a PD consistent with the Global Corporate Default reports produced by Standard & Poor's (S&P) over the period 1981-2015.[17]

Bank's discount rate r is fixed at 1.8% in order to obtain a contractual loan rate c equal to 2.54%, which is very close to the 2.52% average interest rate of new corporate loans made by euro area banks in the period from January 2010 to September 2016.[18] The PDs and yearly probabilities of migration across our standard and substandard categories are extracted from S&P rating migration data using the procedure described in Appendix B. These probabilities are consistent with the alignment of our standard category (j=1) with ratings AAA to BB in the S&P classification and our substandard category (j=2) with ratings B to C.

Table 1
Calibration of the model without aggregate risk

Banks' discount rate	r	1.8%
Yearly probability of migration 1 → 2 if not maturing	a_{21}	7.37%
Yearly probability of migration 2 → 1 if not maturing	a_{12}	6.29%
Yearly probability of default if rated j=1	PD_1	0.85%
Yearly probability of default if rated j=2	PD_2	7.29%
Loss given default	λ	36%
Average time to maturity if rated j=1	$1/\delta_1$	5 years
Average time to maturity if rated j=2	$1/\delta_2$	5 years
Yearly probability of resolution of NPLs	δ_3	44.6%
Newly originated loans per period (all rated j=1)	e_1	1

In a nutshell, we reduce the 7 × 7 rating-migration probabilities and the seven PDs in S&P data to the 2 × 2 migration probabilities and two PDs that appear in matrix M (equation (3)) by calculating weighted averages that take into account the steady-state composition that the loan portfolio would have under its 7-ratings representation. To achieve this composition, we assume that loans have an average duration of five years (or δ_1=δ_2=0.2), as in Table 1; that they have a BB rating at origination and that they then evolve (through improvements or deteriorations in their credit quality

17 We use reports equivalent to S&P (2016) published in 2003 and 2005-2016.

18 We use the euro area (changing composition), annualised agreed rate/narrowly defined effective rate on euro-denominated loans other than revolving loans and overdrafts, and convenience and extended credit card debt issued by banks to non-financial corporations (see **sdw.ecb.europa.eu/quickview**).

before defaulting or maturing) exactly as in our model, but with the seven non-default rating categories in the original S&P data.

Under these assumptions, we obtain an average yearly PD for our standard and substandard categories of 0.9% and 7.3% respectively. As shown in Table 2, given the composition of the "reduced" steady-state portfolio, the average annual loan default rate equals 1.9%, which is below the average 2.5% PD for non-defaulted corporate exposures that the EBA (2013, Figure 12) reports for the period from the first half of 2009 to the second half of 2012 for a sample of EU banks operating under the IRB approach.

Table 2

Endogenous variables under the no-aggregate-risk calibration

(IRB bank, all variables in percentages)

Yearly contractual loan rate, c	2.54
Steady-state portfolio shares (percentage of total loans)	
Standard loans, $x_1^*/(\sum_{j=1,2,3} x_j^*)$	81.29
Substandard loans, $x_2^*/(\sum_{j=1,2,3} x_j^*)$	15.53
NPLs, $x_3^*/(\sum_{j=1,2,3} x_j^*)$	3.18
Average yearly PD on non-defaulted loans, $(\sum_{j=1,2} PD_j x_j^*)/(\sum_{j=1,2} x_j^*)$	1.88
Average yearly PD total loans, $(\sum_{j=1,2} PD_j x_j^* + x_3^*)/(\sum_{j=1,2,3} x_j^*)$	5.00
Steady-state allowances (percentage of total loans)	
Incurred losses	1.14
One-year expected losses	1.78
Lifetime expected losses	4.64
IFRS 9 allowances	2.67
Stage 1 allowances	0.24
Stage 2 allowances	1.28
Stage 3 allowances	1.14
IRB capital requirement for standard loans, γ_1	7.57
IRB capital requirement for substandard loans, γ_2	12.86
IRB minimum capital requirement (percentage of total loans), $\underline{k}$	8.15
IRB minimum capital requirement + CCB (percentage of total loans), $\overline{k}$	10.70

The LGD parameter λ is set at 36%, which roughly matches the average LGD on corporate exposures that the EBA (2013, Figures 11 and 13) reports for the period from the first half of 2009 to the second half of 2012 for the same sample as above. Finally, we set δ_3 equal to 44.6% in order to produce a steady-state fraction of NPLs consistent with the 5% average PD, including defaulted exposures that the EBA (2013, Figure 10) reports for the earliest period in its study, namely the first half of 2008.[19] This value of δ_3 implies an average time to resolution for NPLs of 2.24 years, which is very close to the estimated 2.42-year average duration of corporate insolvency proceedings across EU countries documented by the EBA (2016, Figure 13).

Finally, the assumed flow of newly originated loans, e_1=1, only provides a normalisation and solely affects the size of the steady-state loan portfolio.

The second section of Table 2 reports the volume of credit impairment allowances in steady state using each of the measurement methods that we compare. The third section the IRB capital requirements, the implied overall minimum capital requirement ($\underline{k}$) and the minimum requirement plus CCB ($\overline{k}$) that we use to model the dynamics of CET1.[20] The various impairment measures are clearly ranked, with sizeable differences between them. The steady-state level of EL_t^{IFRS9} is closer to that of EL_t^{1Y} than it is to that of EL_t^{LT} because the steady-state portfolio contains a fairly small (15.5%) fraction of substandard loans ("stage 2" loans under IFRS 9).

The following thought experiment represents a first look into the implications of the model for how various credit impairment measures respond to shocks that erode the expected credit quality of a loan portfolio. Suppose that the loan portfolio is at its steady-state composition at an initial date t=−1. Suppose also that at t=0 the system is hit by a large, unexpected shock that renders an extra 35% of the standard-quality loans of the previous date substandard (instead of remaining standard for one more period), so that their rating migrations, typically driven by a_{11} and a_{21}, become driven by $a'_{11} = a_{11} - 0.35$ and $a'_{21} = a_{21} + 0.35$ respectively. Formally, this means perturbing m_{11} and m_{21} to $m'_{11} = (1 - \delta_1)(a_{11} - 0.35)$ and $m'_{21} = (1 - \delta_1)(a_{21} + 0.35)$ for one period.

From t=1 onwards, the system simply follows its own dynamics, according to the parameters described in Table 1, without further shocks. Note, however, that the presence of an abnormally high amount of substandard loans makes the effects of the initial shock persistent over time. This can be seen in Panel A of Figure 2, which depicts the evolution of NPLs in this thought experiment.

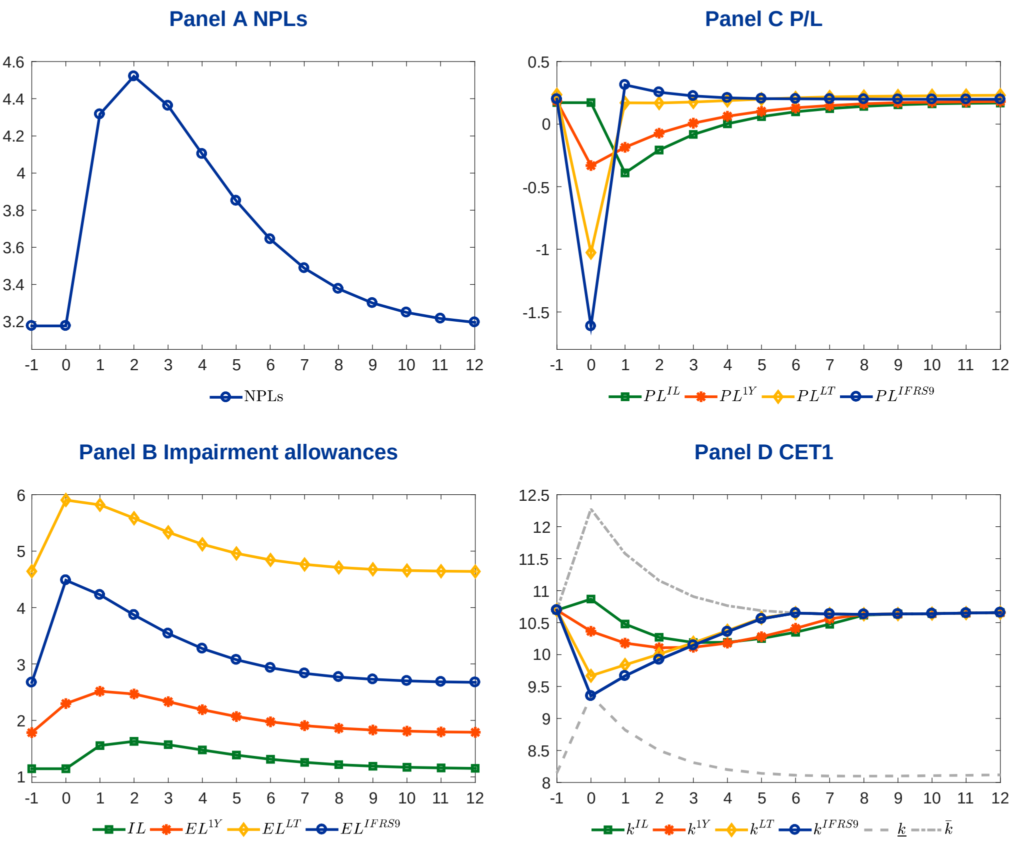
[19] We take this observation, made just before the full negative impact of the global financial crisis was felt, to be the best proxy in the data for the model's "steady state". As shown in Table 2, with this procedure, we obtain a 3.2% share of defaulted exposures in the steady-state portfolio, which is between the 2.5% and 4.4% reported by the EBA (2013, Figure 8) for corporate loans in the second half of 2008 and the first half of 2009 respectively.

[20] To keep the analysis focused, we first discuss the case of IRB banks, postponing the comparison with SA banks until Section 6.

Figure 2

Effects of a negative shock to credit quality

(responses to an unexpected once-and-for-all shock to credit quality; IRB bank, as a percentage of initial exposures)



The results regarding the evolution of the various impairment measures over the same time span are shown in Panel B of Figure 2. Credit loss allowances IL_t, EL_t^{1Y}, EL_t^{LT} and EL_t^{IFRS9} are reported as a percentage of the total initial loans. The levels of the series at t=−1 reflect the different sizes of the various measures in steady state.

The results shown in Figure 2 for t=0,1,2,... are equivalent to a typical impulse response function in macroeconomic analysis. When the shock hits at t=0, all measures except IL_t, which, given its backward-looking nature, reacts with a delay of one period, move upwards for one or two periods before entering a pattern of exponential decay, driven by maturity, defaults, migration of substandard loans back to the standard category and the continued origination of new standard-quality loans.[21]

The responses of EL_t^{1Y} and, when it comes, IL_t to the shock are much smaller than those of the other forward-looking measures. Interestingly, the on-impact response of EL_t^{IFRS9} (which increases by about 1.9 percentage points of initial exposures) exceeds that of EL_t^{LT} (which increases by about

[21] Variations of the experiment that simultaneously shut down or reduce origination of new loans for a number of periods can be easily performed without losing consistency. Experiencing lower loan origination after t=0 delays the process of reversion to the steady state, but does not qualitatively affect the results.

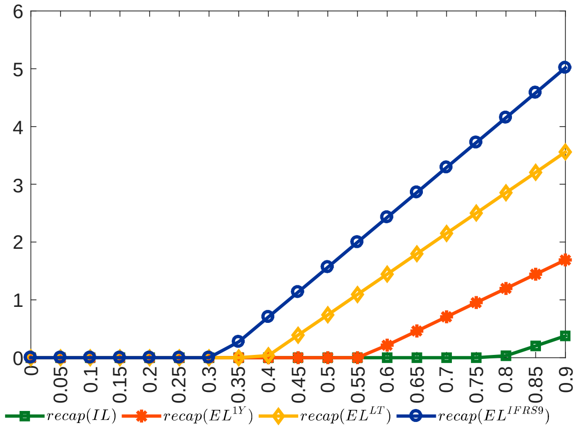
1.3 percentage points). By contrast, EL_t^{1Y} increases by barely 0.5 percentage point at its peak (at t=1) and IL_t increases by roughly 0.4 percentage point at its peak (at t=2).

The implications of the various impairment measures for P/L are described in Panel C of Figure 2. Essentially, each measure spreads the (same final) impact of the shock on P/L over time in a different manner. EL_t^{IFRS9} and, to a lesser extent, EL_t^{LT} front-load the impact of the shock to the extent that P/L is very negative on impact, but then positive and even above normal for a number of periods afterwards. With EL_t^{1Y} (and IL_t), P/L is affected far less (and with a delay) on impact but remains negative for several periods. Interestingly, the measure which allows P/L to return to normal the soonest in this experiment is EL_t^{LT}.

Figure 3

Recapitalisation needs and the size of the shock

(x-axis: fraction of standard loans abnormally turning substandard; y-axis: sum of recapitalisation needs; IRB bank, as a percentage of initial exposures)



Panel D of Figure 2 shows the implications for an IRB bank's CET1. Before the shock hits, at t=−1, the bank is assumed to have a fully-loaded CCB, implying a buffer on top of the minimum required capital of more than 2.5% of total assets. The change in the bands $\underline{k}$ and $\overline{k}$ reflected in the figure are the result of the change in RWAs following the deterioration in the composition of the bank's loan portfolio. The differences in the effects of the alternative measures on CET1 are dramatic, essentially mirroring their impact on P/L.

In the case of IFRS 9, an abnormal extra shift of 35% of the loans from j=1 to j=2 at t=0 implies using the CCB in that very year and having to raise a (small) amount of new equity. Using the alterative measures, no equity issuance is required and the return to normal capital levels occurs solely via earnings retention.

Of course, whether or not there is a need for recapitalisation under the various impairment measures in this thought experiment depends on the ad hoc size of the initial shock, so far fixed at 35% for purely illustrative reasons. However, the (weak) order of the recapitalisation needs that each measurement method would imply happens to be invariant to the size of the shock. This can be seen in Figure 3, which shows the cumulative capital issuance needs implied by a shock like this under each measure (vertical axis) as a function of the additional fraction of standard loans that the shock renders substandard (horizontal axis).

Section 5
Adding aggregate risk

The most natural way to incorporate aggregate risk in the model is to consider an aggregate state variable, s_t, the evolution of which affects the key parameters governing portfolio dynamics and credit losses in the model. To keep things simple, we will assume that s_t follows a Markov chain with two states s=1,2 and time-invariant transition probabilities $p_{s's} = \mathrm{Prob}(s_{t+1} = s' \mid s_t = s)$. In this representation, s=1 could, for example, refer to an expansion or quiet periods, while s=2 could refer to a contraction or crisis periods.[22]

In Appendix B, we extend the model and the formulae for the calculation of portfolio dynamics and impairment allowances to accommodate the case in which the parameters determining the (expected) maturity of the loans, their default probabilities, their migration across ratings, their probability of being resolved when in default, their loss rates upon resolution and the origination of new loans between any dates t and $t+1$ may vary with the arrival state s_{t+1}.

An approach that allows us to keep the analysis recursive as in the baseline model is to expand the vectors describing loan portfolios so that components describe "loans originated in state z, currently in state s and rated j", for each possible (z, s, j) combination, instead of just "loans rated j". In parallel, we expand the transition matrices describing the dynamics of these portfolios to reflect the possible transitions of the aggregate state and their impact on all the relevant parameters. The need to keep track of the state at origination z comes from the need to discount the future credit losses of each loan using the effective contractual interest rate, which now varies with the aggregate state at origination and is denoted by c_z.

5.1 Calibration with aggregate risk

Table 3 shows the calibration of the model with aggregate risk. As explained further in Section A.3 of Appendix A, we allow for state variation in the probabilities of loans migrating across rating categories and into default in a way consistent with the historical correlation between those variables (as observed in S&P rating-migration data) and the US business cycle as dated by the National Bureau of Economic Research (NBER).[23] The dynamics of the aggregate state as parameterised in Table 3 imply that the average duration of an expansion and a contraction is 6.75 years and 2 years respectively, meaning that the system spends about 77% of the time in state s=1. Expansions are characterised by significantly smaller PDs among both standard and substandard loans than contractions. During a contraction, the probability of standard loans being downgraded (or, under IFRS 9, moved into stage 2) is almost double than during an expansion and the probability of substandard loans recovering to standard quality (or returning to stage 1) is reduced by about one-third.

To keep the potential sources of cyclical variation under control, we maintain the parameters determining the effective maturity of performing loans, the speed of resolution of NPLs, the LGD, and the flow of entry of new loans as time invariant (and equal to their values in the calibration without aggregate risk).

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22 For an empirical ratings-migration model in which macroeconomic conditions are represented in this manner, see Bangia, A. et al. (2002).

23 See **www.nber.org/cycles**.

Table 3
Calibration of the model with aggregate risk

Parameters without variation with the aggregate state			
Banks' discount rate	r	1.8%	
Persistence of the expansion state (s=1)	p_{11}	0.852	
Persistence of the contraction state (s=2)	p_{22}	0.5	

Parameters without variation with the aggregate state		If $s' = 1$	If $s' = 2$
Yearly probability of migration 1 → 2 if not maturing	a_{21}	6.16%	11.44%
Yearly probability of migration 2 → 1 if not maturing	a_{12}	6.82%	4.47%
Yearly probability of default if rated j=1	PD_1	0.54%	1.91%
Yearly probability of default if rated j=2	PD_2	6.05%	11.50%
Loss given default	λ	36%	36%
Average time to maturity if rated j=1	$1/\delta_1$	5 years	5 years
Average time to maturity if rated j=2	$1/\delta_2$	5 years	5 years
Yearly probability of resolution of NPLs	δ_3	44.6%	44.6%
Newly originated loans per period (all rated j=1)	e_1	1	1

5.2 Cyclicality of the various impairment measures

Table 4 reports unconditional means, standard deviations and means conditional on each aggregate state for a number of endogenous variables. The variation in the aggregate state causes a significant variation in the composition of the bank's loan portfolio. Not surprisingly, in the contraction state, stage 2 and stage 3 loans represent a larger share of the portfolio, and the overall realised default rate is more than double than in the expansion state. As mentioned in previous sections, the focus of the analysis of the implications for CET1 has thus far been on the case of IRB banks, leaving the comparison with the case of SA banks to Section 6.

The mean relative sizes of the various impairment allowances are essentially the same as obtained for the case without aggregate risk. Interestingly, impairments measured under IFRS 9 are the most volatile, followed closely by those measured under the lifetime expected approach. The least volatile measure is IL.

The decomposition by stage shown for IFRS 9 reveals that allowances associated with NPLs, followed by those associated with stage 2 loans, are those that contribute most to cross-state variation in impairment allowances. However, stage 3 loans are treated in the same way by all measures, which means that the differing volatilities of the various measures must stem from the treatment of stage 1 loans (the same applies across EL^{1Y}, EL^{LT} and EL^{IFRS9}, but is different in IL) and stage 2 loans (the same applies across EL^{LT} and EL^{IFRS9}, but is different in IL and EL^{1Y}) or from the cyclical shift of loans from stage 1 to stage 2 (under EL^{IFRS9}).

Table 4
Endogenous variables under the aggregate risk calibration

(IRB bank, percentage of mean exposures unless otherwise indicated)

	Mean	Standard. deviation	Conditional means	
			s=1	s=2
Yearly contractual loan rate c (%)			2.52	2.62
Share of standard loans (%)	81.35	3.48	82.68	76.85
Share of substandard loans (%)	15.46	1.90	14.59	18.42
Share of NPLs (%)	3.19	1.05	2.73	4.73
Realised default rate (% of performing loans)	1.89	0.90	1.36	3.43
Impairment allowances:				
Incurred losses	1.15	0.38	0.98	1.70
One-year expected losses	1.79	0.50	1.55	2.60
Lifetime expected losses	4.65	0.59	4.36	5.63
IFRS 9 allowances	2.67	0.62	2.38	3.66
Stage 1 allowances	0.24	0.05	0.22	0.33
Stage 2 allowances	1.28	0.21	1.18	1.63
Stage 3 allowances	1.15	0.38	0.98	1.70
IRB minimum capital requirement	8.15	0.07	8.14	8.19
IRB minimum capital requirement + CCB	10.69	0.09	10.68	10.74

5.3 Impact on the cyclicality of P/L and CET1

Table 5 summarises the impact of the various impairment measurement approaches on P/L and CET1 in the case of an IRB bank. Confirming what one might expect after observing the volatility ranking of the impairment measures in Table 4: P/L is significantly more volatile under the more forward-looking EL^{LT} and EL^{IFRS9} than under EL^{1Y}, or IL. EL^{IFRS9} (IL) is clearly the impairment measure producing a higher (lower) volatility of P/L across aggregate states.

The more forward-looking impairment measures are those that make the bank, on average, more CET1-rich in expansion states and less CET1-rich in contraction states; i.e. those that render CET1 more procyclical in this sense. In any case, the reported quantitative differences are not huge, in part because under our assumptions on the bank's management of its CET1, the range of variation in CET1 under any of the impairment measures is limited by the regulation-determined bands of the sS-rule described in equations (21) and (22). As explained above, the bank adjusts its CET1 to remain within those bands by paying dividends or raising new equity.

Thus, a complementary way to assess the potential procyclicality associated with each impairment measure is to look at the frequency and size (conditional on them being strictly positive) of dividends and recapitalisations. Quite intuitively, under all measures, we ascertain that dividend distributions only occur (if at all) during periods of expansion, while recapitalisations only occur (if at all) during periods of contraction.

Relative to EL^{1Y}, the usage of EL^{IFRS9} implies an increase, from 12% to 15%, in the probability that the bank needs to be recapitalised during periods of contraction (mirrored by a more modest increase, from 67% to 70%, in the probability of dividends being paid during periods of expansion).[24]

Table 5

Endogenous variables under the aggregate risk calibration

(IRB bank, percentage of mean exposures unless otherwise indicated)

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
P/L				
Unconditional mean	0.16	0.17	0.23	0.19
Conditional mean (s=1)	0.35	0.41	0.49	0.46
Conditional mean (s=2)	-0.46	-0.61	-0.66	-0.71
Standard deviation	0.34	0.43	0.51	0.50
CET1				
Unconditional mean	10.20	10.19	10.25	10.17
Conditional mean (s=1)	10.38	10.43	10.53	10.46
Conditional mean (s=2)	9.55	9.32	9.28	9.16
Standard deviation	0.76	0.76	0.71	0.77
Probability of dividends being paid (%)				
Unconditional	49.53	51.79	56.38	53.93
Conditional (s=1)	64.20	67.11	73.07	69.89
Conditional (s=2)	0	0	0	0
Dividends, if positive				
Conditional mean (s=1)	0.35	0.36	0.42	0.38
Conditional mean (s=2)	-	-	-	-
Probability of having to recapitalise (%)				
Unconditional	2.34	2.86	2.34	3.41
Conditional (s=1)	0	0	0	0
Conditional (s=2)	10.26	12.50	10.22	14.94
Recapitalisation, if positive				
Conditional mean (s=1)	-	-	-	-
Conditional mean (s=2)	0.42	0.40	0.34	0.38

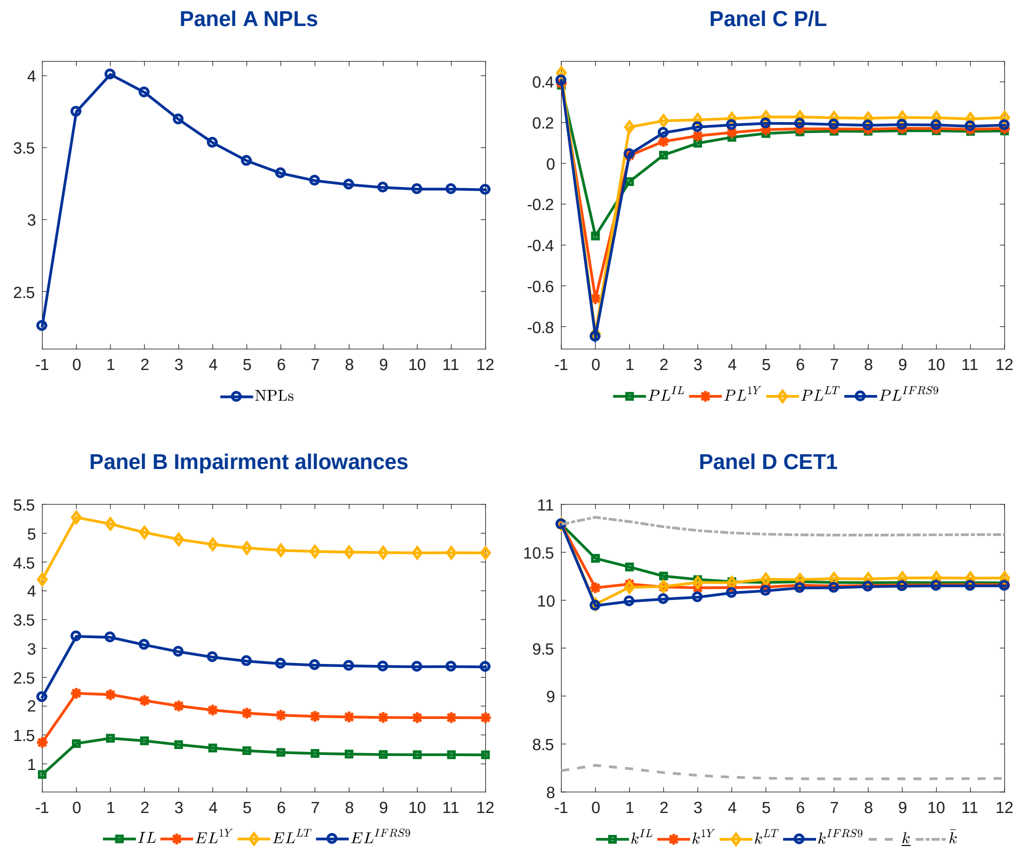
24 However, these effects become counterbalanced by the fact that, when strictly positive, the average size of the recapitalisations needed (and dividends paid) under EL^{IFRS9} is slightly lower than that under EL^{1Y}.

5.4 Effects of the arrival of a contraction

Using the same layout as in Figure 2, Figure 4 shows the effects of the arrival of a contraction at t=0 (i.e. the realisation of s_0=2) having spent a long enough period in the expansion state (i.e. having had s_t=1 for enough dates prior to t=0). From t=1 onwards, the aggregate state follows the Markov chain calibrated in Table 3, thus making the trajectories followed by the variables depicted in Figure 4 stochastic. The figure depicts the average trajectories resulting from simulating 10,000 paths.

Figure 4
Effects of the arrival of a contraction

(average responses to the arrival of s=2 after a long period in s=1; IRB bank, as a percentage of average exposures)



The fact that the trajectories depicted are average trajectories is important for interpreting Figure 4 correctly. For example, in Panel D, the average trajectory of CET1 lies within the average bands of the sS-rule that determines its management, but this does not mean that the bank does not need to recapitalise (or does not pay dividends) after the initial shock. In fact, most of the actual trajectories are either upward and reach the upper band for paying dividends (e.g. if the contraction ends and does not return) or downward and force the bank to recapitalise (e.g. if the contraction lasts a long time or another contraction follows soon after an initial recovery).

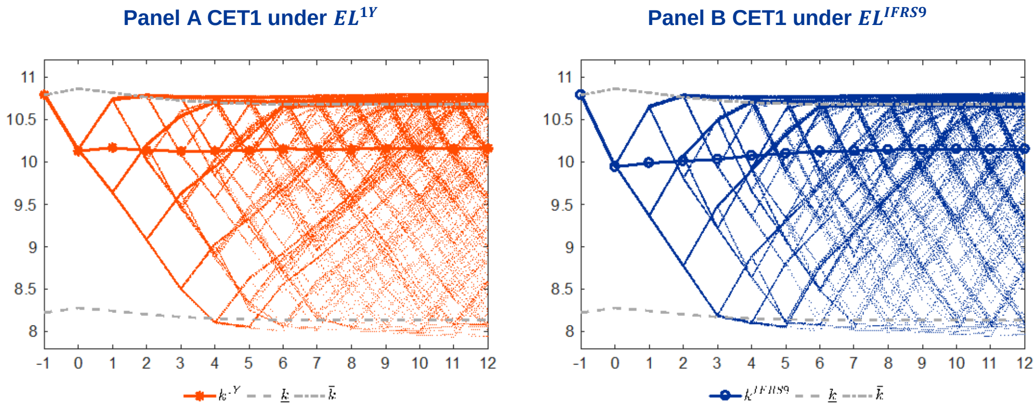
To illustrate the difference between the average trajectory and the realised trajectories , Figure 5 shows 500 simulated trajectories for CET1 under EL^{1Y} and EL^{IFRS9}. Under the current calibration, it takes four consecutive years in the contraction state (s=2) for a bank to use up its CCB and require recapitalisation under IFRS 9. By contrast, under the one-year expected loss approach, the CCB would be used up only after five years in the contraction state.

Intuitively, the closer the average trajectory for CET1 is to the lower band in Panel D of Figure 4, the more likely it is that the bank will need to raise new equity in the course of its recovery from the shock. Thus, as reported in Table 4, the probability of the bank needing to be recapitalised after the shock is higher under EL^{IFRS9} than under any of the other three aproaches.

Figure 5

CET1 after the arrival of a contraction (IRB bank)

(500 simulated trajectories of CET1 under EL^{1Y} and EL^{IFRS9} in response to the arrival of s=2 after a long period in s=1; IRB bank, as a percentage of average exposures)



Section 6
The case of SA banks

Capital requirements for banks following the standardised approach (SA banks) apply to exposures net of specific provisions and, hence, are sensitive to how those provisions are computed. Thus, Table 6 includes the same variables as in Table 5 for IRB banks together with details on the minimum capital requirement implied by each of the impairment measurement methods. Except for the minimum capital requirement and the implied size of a fully-loaded CCB, all the other variables in Table 4 are equally valid for IRB and SA banks.

The results in Table 6 are qualitatively very similar to those described for an IRB bank in Table 5, with some quantitative differences that are worth commenting on. It turns out that, in our calibration, an SA bank holding exactly the same loan portfolio as an IRB bank would be able to support it with somewhat lower average levels of CET1 (between 48 basis points and 157 basis points lower, depending on the impairment measurement method). Therefore, in a typical year, our SA bank features de facto slightly higher leverage levels, and hence slightly higher interest expenses than its IRB counterpart. This explains why its P/L is slightly lower than that of an IRB bank. This difference explains most of the level differences which can be seen in the remaining variables in Table 6.

When comparing impairment measurement methods in the case of an SA bank, the differences are very similar to those observed in Table 5 for IRB banks. The higher state-dependence of the more forward-looking measures explains the higher cross-state differences in CET1, dividends and probabilities of needing capital injections under such measures. As for IRB banks, the differences associated with IFRS 9 relative to either the incurred loss approach or the one-year expected loss approach are significant, but not huge.

To facilitate the comparison of the relevant differences between SA banks and IRB banks, Table 7 contains a selection of variables from Tables 4, 5 and 6. The selection is based on the assumption that the relevant impairment allowances for an SA bank prior to the adoption of IFRS 9 are those of the incurred loss method, IL, while for an IRB bank the one-year expected loss method, EL^{1Y}, is applied. The results point to IFRS 9 having an extremely similar quantitative impact on SA banks and IRB banks, in terms of both the means and the cyclical sensitivity of the relevant variables.

This is further confirmed by Figure 6, which shows the counterpart of Figure 5 for a bank operating under the SA approach. It depicts 500 simulated trajectories for CET1 under IL and EL^{IFRS9}. As in Figure 5, it takes four consecutive years in the contraction state (s=2) for an SA bank under IFRS 9 to use up its CCB and require recapitalisation, while under the incurred loss method, the CCB would be used up only after (roughly) five years in the contraction state.[25]

[25] In this case, the dashed lines that de-limit the band within which CET1 evolves are averages across simulated trajectories, since the relevant sizes of the minimum capital requirement and the minimum capital requirement plus a fully-loaded CCB depend on the size of the corresponding allowances.

Table 6

P/L, CET1, dividends and recapitalisations under SA capital requirements

(SA bank, as a percentage of mean exposures unless otherwise indicated)

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
P/L				
Unconditional mean	0.15	0.16	0.20	0.17
Conditional mean (s=1)	0.34	0.39	0.46	0.44
Conditional mean (s=2)	-0.46	-0.62	-0.69	-0.73
Standard deviation	0.34	0.43	0.51	0.50
Minimum capital requirement				
Unconditional mean	7.72	7.57	6.88	7.36
Conditional mean (s=1)	7.72	756	6.88	7.35
Conditional mean (s=2)	7.74	7.58	6.89	7.37
Standard deviation	0.14	0.17	0.18	0.19
CET1				
Unconditional mean	9.70	9.50	8.68	9.23
Conditional mean (s=1)	9.88	9.76	8.97	9.54
Conditional mean (s=2)	9.04	8.61	7.67	8.19
Standard deviation	0.83	0.83	0.77	0.85
Probability of dividends being paid (%)				
Unconditional	51.32	52.95	59.08	53.20
Conditional (s=1)	66.53	68.64	76.59	68.96
Conditional (s=2)	0	0	0	0
Dividends, if positive				
Conditional mean (s=1)	0.32	0.33	0.35	0.35
Conditional mean (s=2)	-	-	-	-
Probability of having to recapitalise (%)				
Unconditional	2.36	2.67	2.67	2.94
Conditional (s=1)	0	0	0	0
Conditional (s=2)	10.33	11.70	11.68	12.88
Recapitalisation, if positive				
Conditional mean (s=1)	-	-	-	-
Conditional mean (s=2)	0.40	0.30	0.36	0.40

Table 7
SA banks vs IRB banks: highlighted differences

(as a percentage of mean exposures unless otherwise indicated)

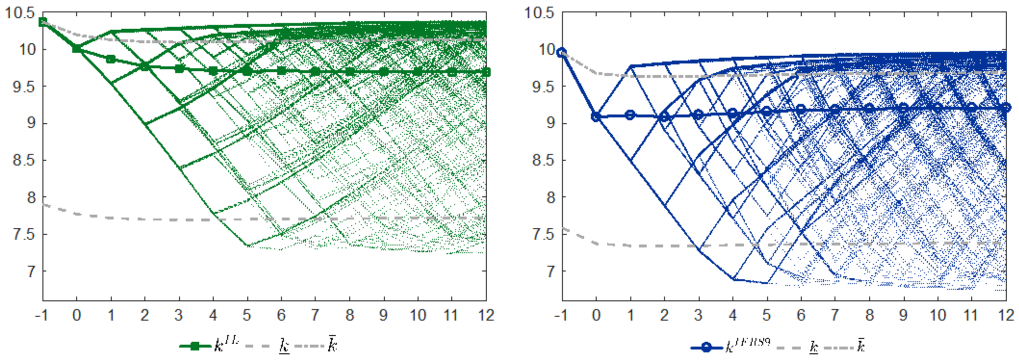
	SA bank		IRB bank	
	IL	EL^{IFRS9}	EL^{1Y}	EL^{IFRS9}
P/L				
Unconditional mean	0.15	0.17	0.17	0.19
Standard deviation	0.34	0.50	0.43	0.50
Minimum capital requirement				
Unconditional mean	7.72	7.36	8.15	8.15
Standard deviation	0.14	0.19	0.07	0.07
CET1				
Unconditional mean	9.70	9.23	10.19	10.17
Standard deviation	0.83	0.85	0.76	0.77
Probability of dividends being paid (%)				
Unconditional	51.32	53.20	51.79	53.93
Conditional on s=1	66.53	68.96	67.11	69.89
Dividends, if positive				
Mean conditional on s=1	0.32	0.35	0.36	0.38
Probability of having to recapitalise (%)				
Unconditional	2.36	2.94	2.86	3.41
Conditional on s=2	10.33	12.88	12.50	14.94
Recapitalisation, if positive				
Mean conditional on s=2	0.40	0.40	0.40	0.38

Figure 6

CET1 after the arrival of a contraction (SA bank)

(500 simulated trajectories of CET1 under IL and EL^{IFRS9} in response to the arrival of s=2 after a long period in s=1; SA bank, as a percentage of average exposures)

Panel A CET1 under IL **Panel B CET1 under EL^{IFRS9}**



Section 7
Extensions

7.1 Particularly severe crises

In this section, we explore whether the severity of crises and the potential anticipation of a particularly severe crisis makes a difference in terms of the assessment of the impairment measurement under IFRS 9 vis-a-vis less forward-looking measures. To keep our graphs readable, they focus on IRB banks and compare the IFRS 9 approach with just one of the alternatives, namely the one-year expected loss approach, which, under our formulation, is similar to the current regulatory approach to loan-loss provisioning for IRB banks.

7.1.1 Unanticipatedly long crises

We first explore what happens with the dynamic responses analysed in the benchmark calibration with aggregate risk when we condition them on the realisation of the contraction state s=2 for four consecutive periods, starting from t=0. Thus, as in the analysis shown in Figure 4, we assume that the bank starts at $t=-1$ with the portfolio and impairment allowances resulting from having been in the expansion state for a long enough period (s=1), and that at t=0 the aggregate state switches to a contraction (s=2).

In Figure 7, we compare the average response trajectories already shown in Figure 4 (where, from t=1 onwards, the aggregate state evolves stochastically according to the Markov chain calibrated in Table 3) with trajectories conditional on remaining in state s=2 for at least up to date t=3 (four years).[26]

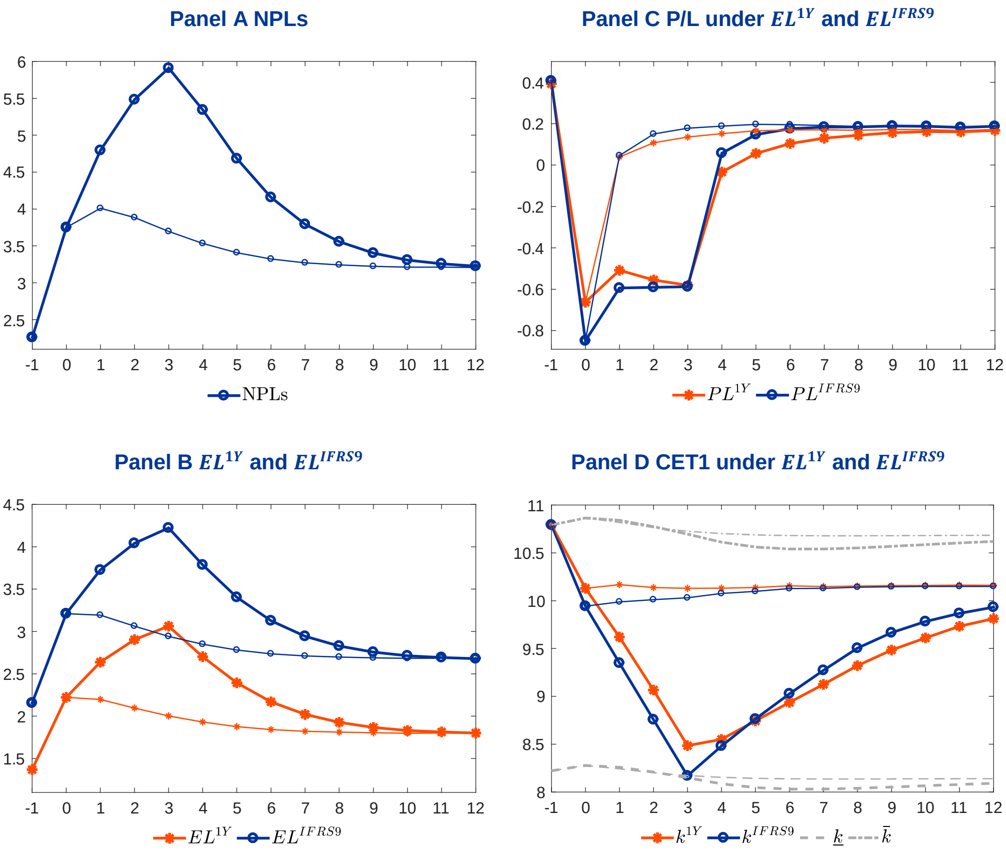
When a crisis is longer than expected, the largest differential impact of EL^{IFRS9} relative to EL^{1Y} still happens in the first year of the crisis (t=0), as EL^{IFRS9} frontloads the expected beyond-one-year losses of the stage 2 loans. In years two to four of the crisis (t=1,2,3), the differential impact of IFRS 9 (compared with one-year) expected losses on P/L lessens before it switches sign (after t=5). In the first years of the crisis, EL^{IFRS9} leaves CET1 closer to the recapitalisation band and in the fourth year (t=3) the duration of the crisis forces the bank to recapitalise only under EL^{IFRS9}. However, EL^{IFRS9} supports a quicker recovery of profitability and, hence, CET1 after t=5.

[26] In the conditional trajectories, the aggregate state is again assumed to evolve from t=4 onwards, according to the calibrated Markov chain.

Figure 7

Unanticipatedly long crises

(average responses to the arrival of s=2 when the contraction is unanticipatedly "long" (thick lines) rather than "average" (thin lines); IRB bank, as a percentage of average exposures)



7.1.2 Anticipatedly long crises

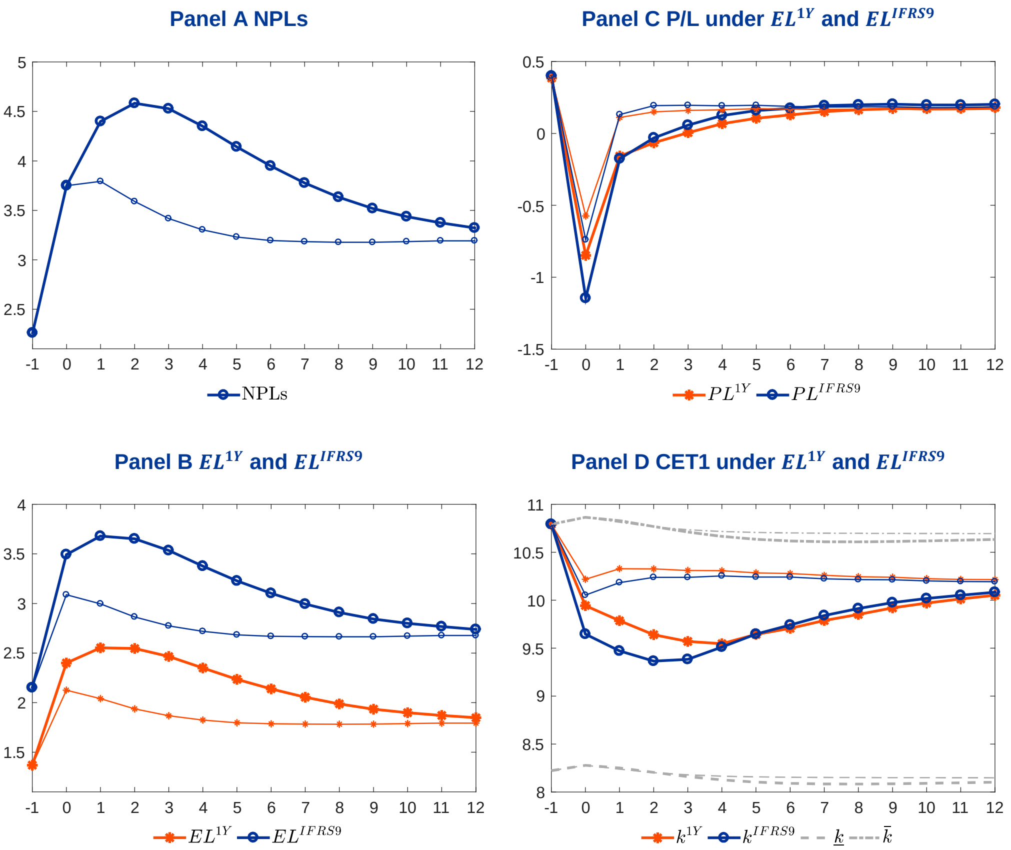
We now turn to the case in which crises can be anticipated as being long from the outset. To study this case, we extend the model to add a third aggregate state that describes "long crises" (s=3) as opposed to "short crises" (s=2) or "expansions" (s=1). To streamline the analysis, we make s=2 and s=3 have exactly the same impact on credit risk parameters as prior s=2 in Table 3 and keep the impact of s=1 on credit risk parameters also exactly the same as in Table 3. The only difference between states s=2 and s=3 is their persistence, which determines the average time it takes for a crisis period to come to an end. Specifically, we consider the following transition probability matrix for the aggregate state:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8520 & 0.6348 & 0.250 \\ 0.1221 & 0.3652 & 0 \\ 0.0259 & 0 & 0.750 \end{pmatrix} \tag{26}$$

which implies an average duration of four years for long crises (s=3), 1.6 years for short crises (s=2) and the same duration as in our benchmark calibration for periods of expansion (s=1). The parameters in (26) are calibrated to make s=3 occur with an unconditional frequency of 8% (equivalent to suffering an average of two long crises per century) and to keep the unconditional frequency of s=1 at the same 77% as in our benchmark calibration.

Figure 8
Anticipatedly long crises

(average responses to the arrival of a contraction at t=0 when it is anticipated to be "long" (s'=3, thick lines) rather than "normal" (s'=2, thin lines); IRB bank, as a percentage of average exposures)



In Figure 8, we compare the average response trajectories that follow the entry into state s=2 (thin lines) or state s=3 (thick lines) having spent a sufficiently long period in state s=1. Therefore, the figure illustrates the average differences between a "normal" short crisis or a "less frequent" long crisis at t=0. It is important to note that both EL^{1Y} and EL^{IFRS9} behave differently across short and long crises from the very first period, since even the one-year ahead loss projections behind EL^{1Y} factor in the lower probability of a recovery at t=1 under s=3 than under s=2. However, EL^{IFRS9} additionally takes into account the losses further into the future that are associated with stage 2 loans. Hence, the differential rise on impact experienced by EL^{IFRS9} is higher than that experienced by EL^{1Y}. This also explains a larger differential initial impact on P/L and CET1. As a result, at the onset of an anticipatedly long crisis EL^{IFRS9} pushes CET1 closer to the recapitalisation band and the difference with regard to EL^{1Y} increases. Quantitatively, however, the effect on CET1 is still moderate, using up on impact less than half of the fully loaded CCB. However, later on in the long crisis, EL^{IFRS9} results, on average, in a quicker recovery of profitability and CET1 than EL^{1Y}.

As a quantitative summary of the implications of an anticipatedly long crisis, the following table reports the unconditional yearly probabilities of the bank needing equity injections, under each of the impairment measures compared, in the baseline model with aggregate risk and in the current extension:

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
Baseline model	2.34%	2.86%	2.34%	3.41%
Model with anticipatedly long crises	3.28%	3.78%	4.23%	4.52%

7.2 Better foreseeable crises

We now consider the case in which some crises can be foreseen one year in advance. Similar to the treatment of long crises in the previous subsection, we formalise this by introducing a third aggregate state, s=3, which describes normal or expansion states, in which a crisis (transition to state s=2) is expected in the next year with a larger than usual probability. For example, we make s=3 identical to s=1 in all respects (i.e. the way it affects the PD, rating migration probabilities and LGD of the loans, etc.) except in the probability of switching to aggregate state s'=2 in the next year.

To streamline the analysis, we look at the case in which s=3 is followed by s'=1 with probability one and assume that half of the crises are preceded by s=3 (while the other half are preceded, as before, by s=1, which means that they are not seen as coming). Adjusting the transition probabilities to imply the same relative frequencies and expected durations of non-crisis versus crisis periods as the baseline calibration in Table 3, the matrix of state transition probabilities used for this exercise is:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8391 & 0.5 & 0.0 \\ 0.0740 & 0.5 & 1 \\ 0.0869 & 0 & 0 \end{pmatrix}.$$

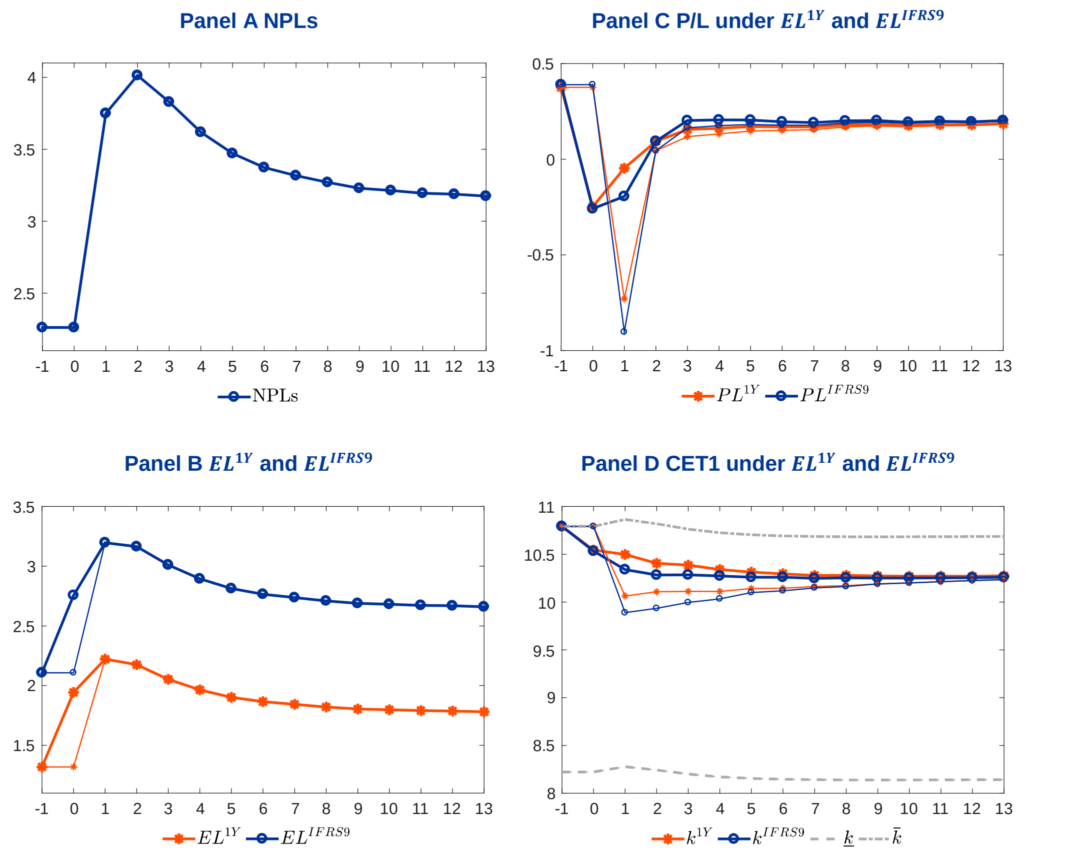
The thick lines in Figure 9 show the average response trajectories to the arrival of the pre-crisis state s'=3 at t=−1 having spent a long time in the normal state s=1. We compare EL^{1Y} and EL^{IFRS9} and include, using thin lines, the results of the baseline model (regarding the arrival of s'=2 at t=0 having been in s=1 for a long period). The results confirm the notion that being able to better anticipate the arrival of a crisis helps to considerably soften its impact on impairment allowances, P/L and CET1.

Finally, as in the previous extension, the following table reports the unconditional yearly probabilities of the bank needing equity injections under each of the impairment measures compared, in the baseline model with aggregate risk and in the current extension. Indeed, crises that are better anticipated imply a lower yearly probability that the bank will need an equity injection:

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
Baseline model	2.34%	2.86%	2.34%	3.41%
Model with better foreseeable crises	1.84%	1.99%	1.54%	2.66%

Figure 9
Better foreseeable crises

(average responses to the arrival of a pre-crisis state at t=−1 after a long period in s=1 (thick lines). Thin lines describe the arrival of s=2 at t=0 in the baseline model; as a percentage of average exposures)



7.3 Other possible extensions

In this section, we briefly describe additional extensions that the model could accommodate at some cost in terms of notational, computational and calibration complexity.

Multiple standard and substandard ratings

Adding more rating categories within the broader standard and substandard categories would essentially imply expanding the dimensionality of the vectors and matrices described in the baseline model and its aggregate-risk extension. If loans were assumed to be originated in more than just one category, the need to keep track of the (various) contractual interest rates for discounting purposes means we would need to expand the dimensionality of the model further. Alternatively, an equivalent and potentially less notationally cumbersome possibility would be to consider the same number of portfolios as different-at-origination loans and to aggregate across them the impairment allowances and the implications for P/L and CET1.

Relative criterion for credit quality deterioration

This extension would be a natural further development of the previous one. Under IFRS 9, the shift to the lifetime approach for a given loan is supposed to be applied not when an “absolute” substandard rating is attained, but when the deterioration in terms of the rating at origination is significant in “relative” terms, for example because the rating has fallen by more than two or three notches. This distinction is relevant if operating under a ratings scale that is finer than the one we have used in our analysis. As in the case with the above-mentioned multiple standard and substandard ratings, keeping the analysis recursive under the relative criterion for treating loans as “stage 1” or “stage 2” loans in IFRS 9 would require considering the same number of portfolios as different-at-origination loan ratings and writing expressions for impairment allowances that impute lifetime expected losses to the components of each portfolio for which the current rating is lower than the initial rating.

Section 8
Macroprudential implications

What are the implications of these results with regard to the potential procyclical effects linked to the various impairment measures? Is the measure associated with IFRS 9 more procyclical than its predecessors? Answering these questions is difficult. Even in the absence of offsetting regulatory filters or sufficient excess capital buffers, a fall in CET1 that reduces the bank's CCB (and hence forces it to cancel its dividend payments), or even leads to it requiring equity issuance in order to continue complying with the minimum capital requirement, does not necessarily imply that the credit supply will contract. It will depend on to what extent banks dislike cancelling dividends and, if the CCB is used up, on how quickly or cheaply the bank can raise new capital. Our simulations are produced as if there were no concerns or imperfections on these two fronts. Otherwise, the bank might be persuaded to reduce its lending. If this process occurs at an economy-wide level (e.g. in response to an aggregate shock), the contractionary effects on aggregate credit supply might be significant, potentially causing negative second-round effects on the system (e.g. by weakening aggregate demand or damaging inter-firm credit chains), ultimately producing larger default rates on surviving loans.

These feedback effects – although theoretically and empirically difficult to assess –are at the heart of the motivation for the macroprudential approach to financial regulation.[27] Similar to discussions on the potential procyclical effects of Basel capital requirements (Kashyap and Stein (2004) and Repullo and Suarez (2013)), there are multiple factors that will determine whether or not IFRS 9 will add procyclicality to the system. For example, even if it causes a contraction in credit supply when a negative shock hits the economy, such a contraction may be lower than the contraction in credit demand, which may also be negatively affected by the shock. Moreover, banks may react to IFRS 9 by choosing to have larger capital buffers in the first place. Besides, the negative effects of an additional contraction in credit supply may be counterbalanced by the advantages of an earlier recognition of loan losses (e.g. by precluding forbearance or the continuation of dividend payments during the initial stages of a crisis), including the possibility that they could enable banks to return to sound financial health more quickly.

Despite all these caveats, recent evidence (including Mésonnier and Monks (2015), Gropp et al. (2016) and Jiménez et al. (2017)) suggests that banks tend to accommodate sudden increases in capital requirements or other regulatory buffers (or, similarly, falls in available regulatory capital) by reducing risk-weighted assets, most typically bank lending, which has a significant impact on the real economy. While the size of the additional procyclical losses of regulatory capital implied by our results is not alarming, it is significant enough to warrant further macroprudential attention.

Fortunately, there is a broad range of policies that may help to address the procyclical effects of IFRS 9 if deemed necessary. One possibility is to rely on the existing regulatory buffers and, specifically, on the countercyclical capital buffer (CCyB), possibly after a suitable revision of its guidance. The national macroprudential authorities could proactively use the CCyB to offset undesirable credit supply effects. This would involve setting the CCyB at a level above zero in expansionary or normal times, so as to have the capacity to partly or fully release it if, and when, the change in aggregate conditions leads to a sudden increase in impairment allowances. This use of this macroprudential tool could be combined with internal and external stress tests as a means to

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[27] As put by Hanson, Kashyap and Stein (2011, p. 5), "in the simplest terms, one can characterise the macroprudential approach to financial regulation as an effort to control the social costs associated with excessive balance sheet shrinkage on the part of multiple financial institutions hit with a common shock."

gauge the importance of the variation in impairment allowances associated with adverse scenarios, guarantee the sufficiency of the micro- and macroprudential buffers, and allow for remedial policy action if required.

Section 9
Concluding remarks

We have described a simple recursive model for the assessment of the level of and cyclical implications of credit impairment loss measurement under IFRS 9. We have calibrated the model to represent a portfolio of corporate loans issued by an EU bank. We have compared the level and dynamic responses to negative shocks of alternative impairment measurement approaches: the current incurred loss approach, the one-year expected loss approach (used to establish the regulatory provisions of IRB banks), the lifetime expected loss approach (which is the one envisaged by the FASB for the United States), and the mixed-horizon expected loss approach of IFRS 9.

Our results suggest that IFRS 9 (and, similarly, the lifetime expected loss approach) will imply more sudden rises in impairment allowances when the cyclical position of the economy switches from expansion to contraction (or if banks experience a shock that sizeably damages the credit quality of their loan portfolios). This implies that P/L and, without the application of regulatory filters, CET1 will decline more sharply at the start of those episodes.

While the early and decisive recognition of forthcoming losses may have significant advantages (e.g. in terms of transparency, market discipline, inducing prompt supervisory intervention, etc.), it may also imply, via its effects on regulatory capital, a loss of lending capacity for banks at the very beginning of a contraction (or in the direct aftermath of a negative credit-quality shock), potentially contributing, through feedback effects, to its severity. With this concern in mind, the quantitative results of this paper suggest that the arrival of an average recession may imply an on-impact loss of CET1 equivalent to one-third of the fully-loaded CCB of the analysed bank. While this loss is larger than under the one-year expected loss approach of the current regulatory provisions for IRB banks, the loss is significantly smaller than the amount that would deplete the fully-loaded CCB, so it can be manageable. Nevertheless, it would be advisable for macroprudential authorities to keep an eye on developments on this front (e.g. by conducting stress tests) and to stand ready to take compensatory measures (e.g. the release of the CCyB), if necessary.

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Appendices

A. Calibration details

A.1 Migration and default rates for our two non-default states

We calibrate the migration and default probabilities of our two non-default loan categories using S&P rating migration data referred to as a finer rating partition. Specifically, we let the 7×7 matrix $\tilde{A}$ describe yearly migrations across the seven non-default ratings in the main S&P classification, namely AAA, AA, A, BBB, BB, B and CCC/C. Under our convention, each element $\tilde{a}_{ij}$ of this matrix denotes a loan's probability of migrating to S&P rating i from S&P rating j, and the yearly probability of default corresponding to S&P rating j can be found as $\widetilde{PD}_j = 1 - \sum_{i=1}^{7} \tilde{a}_{ij}$.[28] We obtain $\tilde{A}$ by averaging the yearly matrices provided by S&P global corporate default studies covering the period from 1981 to 2015:

$$\tilde{A} = \begin{pmatrix} 0.8960 & 0.0054 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0007 \\ 0.0967 & 0.9073 & 0.0209 & 0.0022 & 0.0008 & 0.0006 & 0.0000 \\ 0.0048 & 0.0798 & 0.9161 & 0.0463 & 0.0034 & 0.0026 & 0.0022 \\ 0.0010 & 0.0056 & 0.0557 & 0.8930 & 0.0626 & 0.0034 & 0.0039 \\ 0.0005 & 0.0007 & 0.0044 & 0.0465 & 0.8343 & 0.0618 & 0.0112 \\ 0.0003 & 0.0009 & 0.0017 & 0.0082 & 0.0809 & 0.8392 & 0.1390 \\ 0.0006 & 0.0002 & 0.0002 & 0.0013 & 0.0079 & 0.0432 & 0.5752 \end{pmatrix} \quad (27)$$

which implies

$$\widetilde{PD}^T = (0.0000, 0.0002, 0.0005, 0.0023, 0.0100, 0.0493, 0.2678).$$

In order to calibrate our model, we want to collapse the above seven-state Markov process to the two-state process specified in our model. We want to obtain its 2×2 transition probability matrix, which we denote by A, and the implied probabilities of default in each state, $PD_j = 1 - \sum_{i=1}^{2} a_{ij}$ for j=1,2. To collapse the seven-state process into the two-state process, we assume that the S&P states 1 to 5 (AAA, AA, A, BBB, BB) correspond to our state 1 and S&P states 6 to 7 (B, CCC/C) to our state 2. We also assume that all the loans originated by the bank belong to the BB category, so that the vector representing the entry of new loans in steady state under the S&P classification is $\tilde{e}^T = (0, 0, 0, 0, 1, 0, 0)$. Under these assumptions, we produce an average PD for the steady state portfolio of 1.88%, slightly below the 2.5% average PD on non-defaulted exposures reported by the EBA (2013, Figure 12) for the period from the first half of 2009 to the second half of 2012 for a sample of EU banks following the IRB approach.

The steady state portfolio under the S&P classification can be found as $z^* = [I_{7\times7} - M]^{-1}\tilde{e}$, where the matrix M has elements $\widetilde{m}_{ij} = (1 - \delta_j)\tilde{a}_{ij}$ and δ_j is the independent probability of a loan rated j maturing at the end of period t. For the calibration, we set δ_j=0.20 across all categories, so that loans have an average maturity of five years. The "collapsed" steady state portfolio x^* associated with z^* has $x_1^* = \sum_{i=1}^{5} z_j^*$ and $x_2^* = \sum_{i=6}^{7} z_j^*$.

[28] We have re-weighted the original migration rates in S&P matrices to avoid having "non-rated" as a possible migration.

For the collapsed portfolio, we construct the 3×3 transition matrix M (that accounts for loan maturity) as

$$M = \begin{pmatrix} \frac{\sum_{j=1}^{5}\sum_{i=1}^{5}\widetilde{m}_{ij}z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=1}^{5}\widetilde{m}_{ij}z_j^*}{x_2^*} & 0 \\ \frac{\sum_{j=1}^{5}\sum_{i=6}^{7}\widetilde{m}_{ij}z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=6}^{7}\widetilde{m}_{ij}z_j^*}{x_2^*} & 0 \\ (1-\delta_3/2)PD_1 & (1-\delta_3/2)PD_2 & (1-\delta_3) \end{pmatrix}, \quad (28)$$

where the probabilities of default for the collapsed categories are found as

$$PD_1 = \left(\frac{\sum_{j=1}^{5}\widetilde{PD}_j z_j^*}{x_1^*}\right),$$

and

$$PD_2 = \left(\frac{\sum_{j=6}^{7}\widetilde{PD}_j z_j^*}{x_2^*}\right).$$

To put this in words, we find the moments describing the dynamics of the collapsed portfolio as weighted averages of those of the original distribution, with the weights being determined by the steady state composition of the collapsed categories in terms of the initial categories.

A.2 Calibrating the resolution rate of defaulted loans

The yearly probability of the resolution of NPLs δ_3 is calibrated to match the 5% average probability of default, including defaulted exposures ($PDID$) that the EBA (2013, Figure 10) reports for the second half of 2008. In the model, the value of that probability in steady state can be computed as

$$PDID = \frac{PD_1x_1^*+PD_2x_2^*+x_3^*}{\sum_{j=1}^{3}x_j^*}.$$

Solving for x_3^* we find:

$$x_3^* = \frac{PD_1x_1^* + PD_2x_2^* - (x_1^*+x_2^*)PDID}{PDID-1}. \quad (29)$$

It should be noted that the dynamic system in (1) allows us to compute x_1^* and x_2^* independently from δ_3, so that the law of motion of NPLs evaluated at the steady state implies

$$x_3^* = (1-\delta_3/2)PD_1x_1^* + (1-\delta_3/2)PD_2x_2^* + (1-\delta_3)x_3^*$$

or

$$\delta_3 = \frac{2(PD_1x_1^*+PD_2x_2^*)}{PD_1x_1^*+PD_2x_2^*+2x_3^*}. \quad (30)$$

Finally, we can evaluate (30) using x_1^*, x_2^* and the value of x_3^* found in (29).

A.3 State contingent migration matrices

In the model described in Appendix B, we capture aggregate risk through an aggregate state variable $s_t \in \{1,2\}$ that follows a Markov chain with a time-invariant transition matrix. We calibrate the state contingent migration matrices $M(1)$ and $M(2)$ of such a version of the model following a procedure analogous to that which results in M in (28), but starting from state-contingent versions, $\tilde{A}(1)$ and $\tilde{A}(2)$, of the 7×7 migration matrix $\tilde{A}$ in (27). As described in Section A.1, we can go from each $\tilde{A}(s)$ to the maturity adjusted matrix $\widetilde{M}(s)$ with elements $\widetilde{m}_{ij}(s) = (1-\delta_j)\tilde{a}_{ij}(s)$ and then find the elements of $M(s)$ as weighted averages of the elements of $\widetilde{M}(s)$. To keep things simple, we use the same unconditional weights as in (28), implying

$$M(s) = \begin{pmatrix} \frac{\sum_{j=1}^{5}\sum_{i=1}^{5}\widetilde{m}_{ij}(s)z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=1}^{5}\widetilde{m}_{ij}(s)z_j^*}{x_2^*} & 0 \\ \frac{\sum_{j=1}^{5}\sum_{i=6}^{7}\widetilde{m}_{ij}(s)z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=6}^{7}\widetilde{m}_{ij}(s)z_j^*}{x_2^*} & 0 \\ (1-\delta_3(s)/2)PD_1(s) & (1-\delta_3(s)/2)PD_{2(s)} & (1-\delta_3(s)) \end{pmatrix}$$

where

$$PD_1(s) = \left(\frac{\sum_{j=1}^{5}\widetilde{PD}_j(s)z_j^*}{x_1^*}\right)$$

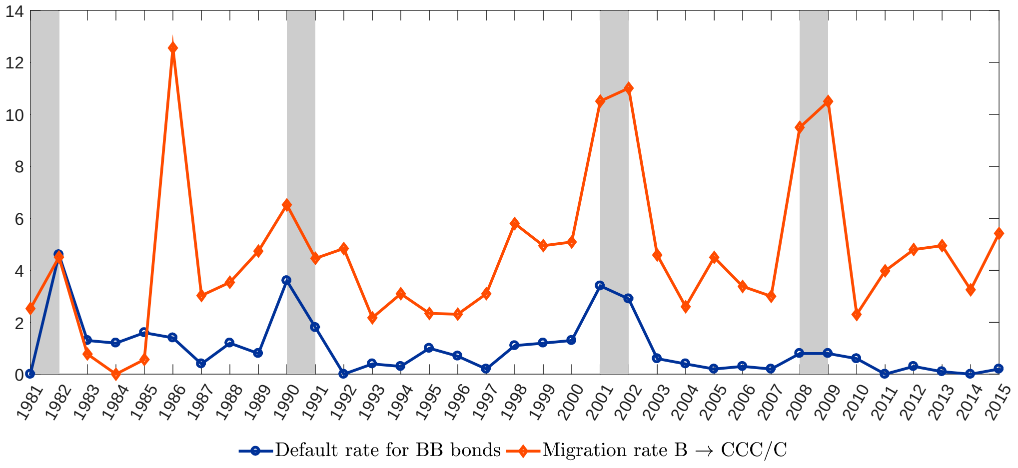
$$PD_2(s) = \left(\frac{\sum_{j=6}^{7}\widetilde{PD}_j(s)z_j^*}{x_2^*}\right)$$

with $\widetilde{PD}_j(s) = 1 - \sum_{i=1}^{7}\tilde{a}_{ij}\ (s)$.

We calibrate $\tilde{A}(1)$ and $\tilde{A}(2)$ exploring the business cycle sensitivity of S&P yearly migration matrices previously averaged to find $\tilde{A}$. We identify state s=1 with normal or expansion years and s=2 with crisis or contraction years. We use the years identified by the NBER as the start of the recession to identify the entry into state s=2 and assume that each of the contractions observed in the period from 1981 to 2015 lasted exactly two years. This is consistent with the NBER dating of US recessions except for the recession which started in 2001, to which the NBER attributes a duration of less than one year. However, the behaviour of corporate rating migrations and defaults around that recession does not suggest, for our purposes, that it was shorter than the other three. To illustrate this, Figure A1 depicts the time series of two of the elements of the yearly default rates $\widetilde{PD}_j$ and migration matrices $\tilde{A}$ whose cyclical behaviour is more evident: (i) the default rate among BB exposures ($\widetilde{PD}_5$) and (ii) the migration rate from a B rating to a CCC/C rating ($\tilde{a}_{7,6}$). The year 2002 stands out as a year in which there was a marked deterioration in credit quality among exposures rated BB and B.

Figure A1

Sensitivity of default and migration rates to aggregate states



Notes: The chart shows selected yearly S&P default and downgrading rates. The grey bars identify two-year periods following the start of NBER recessions.

In the light of this, we estimate $A(2)$ by averaging the yearly migration matrices of years 1981, 1982, 1990, 1991, 2001, 2002, 2008 and 2009, and $A(1)$ by averaging all the remaining ones. This leads to

$$\tilde{A}(1)=\begin{pmatrix} 0.8923 & 0.0057 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0000 \\ 0.1012 & 0.9203 & 0.0209 & 0.0023 & 0.0007 & 0.0003 & 0.0000 \\ 0.0039 & 0.0668 & 0.9228 & 0.0500 & 0.0036 & 0.0025 & 0.0027 \\ 0.0010 & 0.0058 & 0.0495 & 0.8939 & 0.0668 & 0.0036 & 0.0043 \\ 0.0007 & 0.0002 & 0.0040 & 0.0429 & 0.8484 & 0.0679 & 0.0117 \\ 0.0000 & 0.0009 & 0.0020 & 0.0082 & 0.0680 & 0.8511 & 0.1548 \\ 0.0000 & 0.0002 & 0.0001 & 0.0013 & 0.0059 & 0.0360 & 0.5860 \end{pmatrix},$$

implying

$$\widetilde{PD}(1)^T=(0.0000, 0.0001, 0.0002, 0.0014, 0.0063, 0.0386, 0.2405).$$

and

$$\tilde{A}(2)=\begin{pmatrix} 0.9087 & 0.0044 & 0.0003 & 0.0005 & 0.0002 & 0.0000 & 0.0030 \\ 0.0786 & 0.8632 & 0.0209 & 0.0014 & 0.0013 & 0.0017 & 0.0000 \\ 0.0077 & 0.1237 & 0.8936 & 0.0340 & 0.0026 & 0.0027 & 0.0009 \\ 0.0010 & 0.0050 & 0.0767 & 0.8899 & 0.0482 & 0.0028 & 0.0024 \\ 0.0000 & 0.0022 & 0.0057 & 0.0587 & 0.7865 & 0.0411 & 0.0095 \\ 0.0013 & 0.0007 & 0.0008 & 0.0076 & 0.1245 & 0.7988 & 0.0858 \\ 0.0027 & 0.0002 & 0.0006 & 0.0025 & 0.0143 & 0.0676 & 0.5389 \end{pmatrix},$$

implying

$$\widetilde{PD}(1)^T=(0.0000, 0.0005, 0.0014, 0.0054, 0.0224, 0.0853, 0.3596).$$

Finally, we set $p_{12}=\text{Prob}(s_{t+1}=1 \mid s_t=2)$ equal to 0.5 so that contractions have an expected duration of two years, and $p_{21}=\text{Prob}(s_{t+1}=2 \mid s_t=1)$ equal to 0.148 so that expansion periods have the same average duration as the ones observed in our sample period, (35-8)/4=6.75 years.

B. The model with aggregate risk

In this appendix, we present the equations of the benchmark model with aggregate risk. We capture the latter by introducing an aggregate state variable that can take two values $s_t \in \{1,2\}$ at each date t and follows a Markov chain with time-invariant transition probabilities $p_{s's}=\text{Prob}(s_{t+1}=s' \mid s_t=s)$. The approach can be trivially generalised to deal with a larger number of aggregate states.

In order to measure expected losses corresponding to default events at any future date t, we have to keep track of the aggregate state in which the loans existing at t were originated, z=1,2, the aggregate state at time t, s=1,2, and the credit quality or rating of the loan at t, j=1,2,3. Thus, it is convenient to describe (stochastic) loan portfolios held at any date t as vectors in the form

$$y_t = \begin{pmatrix} x_t(1,1,1) \\ x_t(1,1,2) \\ x_t(1,1,3) \\ x_t(1,2,1) \\ x_t(1,2,2) \\ x_t(1,2,3) \\ x_t(2,1,1) \\ x_t(2,1,2) \\ x_t(2,1,3) \\ x_t(2,2,1) \\ x_t(2,2,2) \\ x_t(2,2,3) \end{pmatrix}, \tag{31}$$

where component $x_t(z,s,j)$ denotes the measure of loans at t that were originated in aggregate state z, are in aggregate state s and have rating j.[29]

Our assumptions regarding the evolution and pay-offs of the loans between any date t and t+1 are as follows. Loans rated j=1,2 at t mature at $t+1$ with probability $\delta_j(s')$, where s' denotes the aggregate state at $t+1$ (unknown at date t). In the case of NPLs (j=3), $\delta_3(s')$ represents the independent probability of a loan being resolved, in which case it pays back a fraction $1-\tilde{\lambda}(s')$ of its unit principal and exits the portfolio. Conditional on s', each loan rated j=1,2 at t which matures at $t+1$ defaults independently with probability $PD_j(s')$, being resolved within the period with probability $\delta_3(s')/2$ or entering the stock of NPLs (j=3) with probability $1-\delta_3(s')/2$. Maturing loans that do not default pay back their principal of one plus the contractual interest c_z established at origination.

Conditional on s', each loan rated j=1,2 at t which does not mature at $t+1$ goes through one of the following exhaustive possibilities.

1. Default, which occurs independently with probability $PD_j(s')$, and in which case one of two things can happen: (i) it is resolved within the period with probability $\delta_3(s')/2$; (ii) it enters the stock of NPLs (j=3) with probability $1-\delta_3(s')/2$.
2. Migration to rating $i \neq j$ (i=1,2), in which case it pays interest c_z and continues for one more period; this occurs independently with probability $a_{\mathrm{ij}}(s')$.
3. Continuation in rating j, in which case it pays interest c_z and continues for one more period; this occurs independently with probability

   $$a_{jj}(s') = 1 - a_{ij}(s') - PD_j(s').$$

B.1 Portfolio dynamics under aggregate risk

Under aggregate risk, the dynamics of the loan portfolio between any dates t and $t+1$ is no longer deterministic, but driven by the realisation of the aggregate state variable at $t+1$, s_{t+1}. To describe the dynamics of the system in a compact way, let the binary variable $\xi_{t+1}=1$ if $s_{t+1}=1$ and $\xi_{t+1}=0$ if $s_{t+1}=2$. The dynamics of the system can be described as

$$y_{t+1} = G(\xi_{t+1})y_t + g(\xi_{t+1}),$$

[29] Along a specific history (or sequence of aggregate states), for any z and j, the value of $x_t(z,s,j)$ will equal 0 whenever $s_t \neq s$.

where

$$G(\xi_{t+1}) = \begin{pmatrix} \begin{pmatrix} \xi_{t+1}M(1) & \xi_{t+1}M(1) \\ (1-\xi_{t+1})M(2) & (1-\xi_{t+1})M(2) \end{pmatrix} & 0_{6\times 6} \\ 0_{6\times 6} & \begin{pmatrix} \xi_{t+1}M(1) & \xi_{t+1}M(1) \\ (1-\xi_{t+1})M(2) & (1-\xi_{t+1})M(2) \end{pmatrix} \end{pmatrix},$$

$$g(\xi_{t+1})^T = (\xi_{t+1}e_1(1), 0, 0, 0, 0, 0, 0, 0, 0, (1-\xi_{t+1})e_1(2), 0, 0),$$

$$\xi_{t+1} = \begin{cases} 1 & \text{if } u_{t+1} \in [0, p_{1s_t}] \\ 0 & \text{otherwise,} \end{cases}$$

$$s_{t+1} = \xi_{t+1} + 2(1-\xi_{t+1}),$$

u_{t+1} is an independently and identically distributed, uniform random variable with support [0,1], $e_1(s')$ is the (potentially different across states s') measure of new loans originated at $t+1$, and $0_{6\times 6}$ denotes a 6×6 matrix full of zeros.

B.2 Incurred losses

Incurred losses measured at date t would be those associated with NPLs that are part of the bank's portfolio at date t. Thus, the incurred losses reported at t would be given by

$$IL_t = \sum_{z=1,2} \sum_{s=1,2} \lambda(s) x_t(z, s, 3),$$

where $\lambda(s)$ is the expected LGD on a NPL conditional on being in state s in date t. This can be more compactly expressed as

$$IL_t = \hat{b} y_t. \tag{32}$$

where $\hat{b} = (0, 0, \lambda(1), 0, 0, \lambda(2), 0, 0, \lambda(1), 0, 0, \lambda(2))$.

The expected LGD conditional on each current state s can be found as functions of the previously specified primitives of the model (state-transition probabilities, probabilities of the loans being resolved in subsequent periods, and loss rates $\tilde{\lambda}(s')$ suffered if resolution happens in each of the possible future states s') by solving the following system of recursive equations:

$$\lambda(s) = \sum_{s=1,2} p'_s s \left[\delta_3(s')\tilde{\lambda} + (1-\delta_3(s'))\lambda(s')\right], \tag{33}$$

for s=1,2.

B.3 Discounted one-year expected losses

Based on the loan portfolio held by the bank at t, the allowance computed on the basis of discounted one-year expected losses adds to the incurred losses written above the losses stemming from default events expected to occur within the year immediately following. Since a period in the model is one year, the corresponding allowances are given by

$$EL_t^{1Y} = (b_\beta + \hat{b}) y_t, \tag{34}$$

where $b_\beta = (\beta_1 b, \beta_2 b)$, $\beta_z = 1/(1+c_z)$, and $b = (b_{11}, b_{12}, 0, b_{21}, b_{22}, 0)$, with

$$b_{sj} = \sum_{s=1,2} p_{s's} PD_j(s') \{[\delta_3(s')/2]\tilde{\lambda}(s') + [1 - \delta_3(s')/2]\lambda(s')\}, \tag{35}$$

for j=1,2. The coefficients defined in (35) attribute one-year expected losses to loans rated j=1,2 in state s by taking into account their PD and LGD over each of the possible states s' that can be reached at $t+1$, where the corresponding s' are weighted by their probability of occurring given s. The losses associated with these one-year ahead defaults are discounted using the contractual interest rate of the loans, c_z, as set at their origination. In Section B.6, we derive an expression for the endogenous value of that rate under our assumptions on loan pricing. As for the loans that are already non-performing (j=3) at date t, the term $\hat{b}y_t$ in (34) implies attributing their conditional-on-s LGD to them, exactly as in (32).

B.4 Discounted lifetime expected losses

Allowances computed on a lifetime-expected basis imply taking into account not only the default events that could affect the currently performing loans in the next year, but also those occurring in any subsequent period. Building on prior notation and the same approach explained for the model without aggregate risk, these allowances can be computed as

$$\begin{aligned} EL_t^{LT} &= b_\beta y_t + b_\beta M_\beta y_t + b_\beta M_\beta^2 y_t + b_\beta M_\beta^3 y_t + \ldots + \hat{b} y_t \\ &= b_\beta \left(I + M_\beta + M_\beta^2 + M_\beta^3 + \ldots\right) y_t + \hat{b} y_t \\ &= b_\beta \left(I - M_\beta\right)^{-1} y_t + \hat{b} y_t = \left(b_\beta B_\beta + \hat{b}\right) y_t, \end{aligned} \tag{36}$$

with

$$M_\beta = \begin{pmatrix} \beta_1 M_p & 0_{6\times 6} \\ 0_{6\times 6} & \beta_2 M_p \end{pmatrix},$$

$$M_p = \begin{pmatrix} p_{11} M(1) & p_{12} M(1) \\ p_{21} M(2) & p_{22} M(2) \end{pmatrix},$$

$$M(s') = \begin{pmatrix} m_{11}(s') & m_{12}(s') & 0 \\ m_{21}(s') & m_{22}(s') & 0 \\ (1-\delta_3(s')/2) PD_1(s') & (1-\delta_3(s')/2) PD_2(s') & (1-\delta_3(s')) \end{pmatrix},$$

and $m_{ij}(s') = \left(1-\delta_j(s')/2\right) a_{ij}(s')$.

B.5 Discounted expected losses under IFRS 9

As already mentioned, IFRS 9 adopts a hybrid approach that combines the one-year-ahead and lifetime approaches described above. Specifically, it applies the one-year-ahead measurement to loans whose credit quality has not increased significantly since origination. For us, these are the loans with j=1, namely those in the components $x_t(z,s,1)$ of y_t. By contrast, it applies the lifetime measurement to loans whose credit risk has increased significantly since origination. For us, these are the loans with j=2, namely those in the components $x_t(z,s,2)$ of y_t.

As in the case without aggregate risk, it is convenient to split vector y_t into a new auxiliary vector

$$\hat{y}_t = \begin{pmatrix} x_t(1,1,1) \\ 0 \\ 0 \\ x_t(1,2,1) \\ 0 \\ 0 \\ x_t(2,1,1) \\ 0 \\ 0 \\ x_t(2,2,1) \\ 0 \\ 0 \end{pmatrix},$$

which contains the loans with j=1, and the difference

$$\tilde{y}_t = y_t - \hat{y}_t,$$

which contains the rest.

Combining the formulae obtained in (34) and (36), the impairment allowances under IFRS 9 can be described compactly as[30]

$$EL_t^{IFRS9} = b_\beta \hat{y}_t + b_\beta B_\beta \tilde{y}_t + \hat{b} y_t. \tag{37}$$

B.6 Determining the initial loan rate

Taking advantage of the recursivity of the model, for given values of the contractual interest rates c_z of the loans originated in each of the aggregate states z=1,2, one can obtain the ex-coupon value of a loan originated in state z, when the current aggregate state is s and its current rating is j, $v_j(z,s)$, by solving the system of Bellman-type equations given by

$$v_j(z,s) = \mu \sum_{s'=1,2} p_{s's} \Bigg[\left(1 - PD_j(s')\right) c_z + \left(1 - PD_j(s')\right) \delta_j(s') + PD_j(s') \left(\frac{\delta_3(s')}{2}\right) \left(1 - \tilde{\lambda}(s')\right) + m_{1j}(s') v_1(z,s') + m_{2j}(s') v_2(z,s') + m_{3j}(s') v_3(z,s') \Bigg], \tag{38}$$

for $(z,s,j) \in \{1,2\} \times \{1,2\} \times \{1,2\}$, and

$$v_j(z,s) = \mu \sum_{s'=1,2} p_{s's} \left[\delta_3(s') \left(1 - \tilde{\lambda}(s')\right) + (1 - \delta_3(s')) v_3(z,s') \right],$$

for $(z,s,j) \in \{1,2\} \times \{1,2\} \times \{3\}$.

Under perfect competition and using the fact that all loans are assumed to be of credit quality j=1 at origination, the interest rates c_z can be found as those that make $v(z,z,1) = 1$ for z=1,2 respectively.

30 These definitions clearly imply $EL_t^{IFRS9} = EL_t^{LT} - b_\beta(B_\beta - I)\hat{y}_t \leq EL_t^{LT}$ and $EL_t^{IFRS9} = EL_t^{1Y} + b_\beta(B_\beta - I)\tilde{y}_t \geq EL_t^{1Y}$.

B.7 Implications for P/L and CET1

By trivially extending the formula derived for the case without aggregate risk, the result of the P/L account with aggregate risk can be written as

$$PL_t = \sum_{z=1,2}\left\{\sum_{j=1,2}\left[c_z\left(1-PD_j(s_t)\right)-\frac{\delta_3(s_t)}{2}PD_j(s_t)\tilde{\lambda}(s_t)\right]x_{t-1}(z,s_t,j)-\delta_3(s_t)\tilde{\lambda}(s_t)x_{t-1}(z,s_t,3)\right\}$$
$$-r\left(\sum_{z=1,2}\sum_{j=1,2,3}x_{t-1}(z,s_t,j)-a_{t-1}-k_{t-1}\right)-\Delta a_t, \quad (39)$$

which differs from (20) in terms of dependence on s_t, the aggregate state at the end of period t, a number of the relevant parameters affecting the default, resolution and loss upon resolution of the loans.

With the same logic as in the baseline model, dividends and equity injections are now determined by

$$\text{div}_t = \max\left[(k_{t-1}+PL_t)-1.3125\underline{k}_t, 0\right] \quad (40)$$

and

$$\text{recap}_t = \max\left[\underline{k}_t-(k_{t-1}+PL_t), 0\right]. \quad (41)$$

Finally, for IRB banks, the minimum capital requirement is now given by[31]

$$\underline{k}_t^{IRB} = \sum\nolimits_{j=1,2}\gamma_j(s_t)x_{jt}, \quad (42)$$

with

$$\gamma_j(s_t) = \lambda(s_t)\frac{1+\left[\left(\sum_{s'}p_{s's}\frac{1}{\delta_j(s')}\right)-2.5\right]\overline{m}_j}{1-1.5\overline{m}_j}\left[\Phi\left(\frac{\Phi^{-1}(\overline{PD}_j)+\overline{\text{cor}}_j^{0.5}\Phi^{-1}(0.999)}{\left(1-\overline{\text{cor}}_j\right)^{0.5}}\right)-\overline{PD}_j\right], \quad (43)$$

where $\overline{m}_j = [0.11852-0.05478\ln(\overline{PD}_j)]^2$ is a maturity adjustment coefficient, $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution and cor_j is a correlation coefficient fixed as $\overline{\text{cor}}_j = 0.24-0.12(1-\exp(-50\overline{PD}_j))/(1-\exp(-50))$, and

$$\overline{PD}_j = \sum\nolimits_{i=1,2}\pi_i PD_j(s_i) \quad (44)$$

is the through-the-cycle PD for loans rated j (with π_i denoting the unconditional probability of aggregate state i). Equation (44) assumes that the bank follows a strict through-the-cycle approach to the calculation of capital requirements (which avoids adding cyclicality to the system through this channel).[32]

[31] For SA banks, the equation for the minimum capital requirement in (25) remains valid.

[32] Under a point-in-time approach, $\overline{PD}_j$ in (43) should be replaced by $\overline{PD}_j(s_t) = \sum_{s'}p_{s's_t}PD_j(s')$.

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Jorge Abad
CEMFI; email: **jorge.abad@cemfi.edu.es**

Javier Suarez
ESRB Advisory Scientific Committee and CEMFI; email: **suarez@cemfi.es**

Postal address	60640 Frankfurt am Main, Germany
Telephone	+49 69 1344 0
Website	**www.esrb.europa.eu**

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